MATH 127 Sample Midterm Test 2 Solutions DRAFT

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1. (a) $x + y \leq 5000$
   (b) $1x + 1.2y \leq 5400$
   (c) There are two other implicit contstraints, $x \geq 0$ and $y \geq 0$. [diagram here] The coordinates of the corners of the feasible region are $(0,0)$, $(5000,0)$, $(0,4500)$, and $(3000,2000)$.
   (d) We know from the theory of linear programming that the optimal production scheme must occur at one of the corners of the feasible region. Trying each of them in turn, we have $(0,0)$ generating a profit of $0.15(0)+0.17(0) = 0$ dollars per day, $(5000,0)$ generating a profit of $0.15(5000)+0.17(0) = 750$ dollars per day, $(0,4500)$ generating a profit of $0.15(0)+0.17(4500) = 765$ dollars per day, and $(3000,2000)$ generating a profit of $0.15(3000)+0.17(2000) = 790$ dollars per day. Therefore the maximum profit is 790 dollars per day, occuring when 3000 cartons of regular cola and 2000 cartons of diet cola are produced.

2. We can do parts (a), (b), (c) all at once, with the following chart:

	$(0,0)$	$(5000,0)$	$(0,4500)$	$(3000,2000)$	max
0.15x+0.17y	0	750	765	790	790
0.20x+0.17y	0	1000	765	940	1000
0.15x+0.19y	0	750	855	830	855
0.15x+0.16y	0	750	720	770	770

   Note that if the profit function is $0.20x + 0.17y$, maximum profit is realized if Copsi produces all regular cola; if the profit function is $0.15x + 0.19y$, maximum profit is realized when it produces all diet cola; and if the profit function is $0.15x + 0.16y$, maximum profit is realized when it produces a blend of colas as in the previous problem.

   So, as the profit function changes, the optimal production scheme may vary radically. The explanation for that behaviour is in the following three diagrams: [insert diagrams] As the slope of the line representing the profit function changes the point where the lines first touch the feasible region may jump from one corner to another. Understanding when that might happen is the subject of “sensitivity analysis”, another major topic in linear programming. The critical values are the slopes of the sides of the feasible region.

3. In the second scenario, P dollars becomes $A = P(1+0.062/4)^4$ dollars after one year; that is equivalent to $A = P(1.0635) = P(1+0.0635)$, or an interest rate of 6.35%. The first scenario, at 6.4% per year, is the better investment.

4. In this case, we have $180,000 = 135,000(1+i)^3$. Solving for i we have

$$(1+i)^3 = \frac{180}{135} = 1.3333 \implies 1+i = (1.3333)^{1/3} = 1.1006 \implies i = 10.06\%;$$

   “just over 10%” is a good answer.

5. The key notion in this question is that we are looking at the value of some amount in the future, so it is an ordinary annuity (regular payment R known, future amount A unknown) or a sinking fund (future amount A

known, regular payment R unknown). You can either memorize the ordinary annuity formula, or you can work it out from scratch. Either way, the formula is

$$A = R\left(\frac{(1+i)^n - 1}{i}\right).$$

That formula also applies to sinking funds.

(a) In this case we have $n = 15$, $i = 0.09$, $R = 25,000$, hence

$$A = 25,000\left(\frac{1.09^{15} - 1}{0.09}\right) = 25,000 \times 29.3609 = 734,022.91$$

approximately.

(b) The formula for a sinking fund is exactly the same, except that A is known and R is unknown. In this case, the interest rate is the same and the number of periods is the same, so we can save some work and write directly

$$2,000,000 = R \times 29.3609 \implies R = 68,117.80.$$

The Cooper Foundation will have to save \$68,117.80 per year at 9% compounded annually to have \$2,000,000 at the end of 15 years.

6. The key to this question is that we have a lump of cash now instead of in the future (the car has to be bought with cash now), so it is a case for amortization (present value P known, regular payment R unknown) or present value (regular payment R known, present value P unknown). You can either memorize the formula for amortization, or you can work it out from scratch:

$$P = R\left(\frac{1-(1+i)^{-n}}{i}\right)$$

The formula also applies to the present value of an annuity.

(a) In this case, $P = 12,000$, $i = 0.063/12 = 0.0053$, $n = 3 \times 12 = 36$, and the equation is

$$12,000 = R\left(\frac{1-(1.0053)^{-36}}{0.0053}\right) = R \times 32.4159 \implies R = \frac{12,000}{32.4159} = 370.19$$

approximately. Alice's monthly payment will be about \$370.19.

(b) In this case P is unknown, $R = 425$, and all the other variables are the same, so $P = 425 \times 32.4159 = 13,776.75$. Alice can afford to buy a car costing up to 13,776.75 now.

7. This problem is hard. The technology required is just linear programming, but setting up the problem in the first place is difficult. At first it might seem that there are four unknown variables: x, the number of lamps Regina orders from A; y, the number of lamps Saskatoon orders from A; z, the number of lamps Regina orders from B; and w, the number of lamps Saskatoon orders from B. However, we know that Regina ordered 25 lamps altogether, so $z = 25 - x$ and we can eliminate z; similarly, Saskatoon ordered 20 lamps altogether, so $w = 20 - y$ and we can eliminate w. The inequalities are:

Supplier A limit: $x + y \leq 10$

Supplier B limit: $z + w \leq 40 \implies (25 - x) + (20 - y) \leq 40 \implies 5 \leq x + y.$

In summary, we can write $5 \leq x + y \leq 10$. We also have the implicit constraints $x \geq 0$ and $y \geq 0$. See the sketch of the feasible region [insert drawing].

The corners of the feasible region have (x,y) coordinates $(5,0)$, $(0,5)$, $(10,0)$, and $(0,10)$. We calculate the objective, the shipping cost, for each of those scenarios: $(5,0)$ implies $z = 20$ and $w = 20$ for a total shipping cost of $5x + 6y + 7z + 4w = 5(5) + 6(0) + 7(20) + 4(20) = 245$; $(0,5)$ implies $z = 25$ and $w = 15$ for a total shipping cost of $5(0)+6(5)+7(25)+4(15)=265$; $(10,0)$ implies $z=15$ and $w=20$ for a total shipping cost of $5(10)+6(0)+7(15)+4(20)=235$; and finally, $(0,10)$ implies $z=25$ and $w=10$ for a total shipping cost of $5(0)+6(10)+7(25)+4(10)=275$. Shipping cost is minimized in the $(10,0)$ scenario.

In summary, total shipping costs are minimized when the Regina store orders 10 lamps from A and 15 lamps from B, and the Saskatoon store orders no lamps from A and 20 lamps from B.