Degrees of Separation in MATH 127

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> *I read somewhere that everybody on this planet is separated by only six other people. Six degrees of separation between us and everyone else on this planet. The President of the United States, a gondolier in Venice, just fill in the names. I find it extremely comforting that we're so close. [But] we're so close because you have to find the right six people to make the right connection... I am bound, you are bound, to everyone on this planet by a trail of six people.* — Playwright John Guare

The object of this exercise is to investigate the pattern of acquaintences among students in the MATH 127 class. I gathered data during the lecture on Thursday, January 25, 2006. Names were taken from the official class list, and those who weren't present during the class were removed from this project.

1 Data Gathering

To gather the data, I asked the first student in the list whether she was acquanted with each of the others, and I entered a 1 into the first row of Table 1 if the answer was 'yes', a 0 into the row if the answer was 'no'. Students are represented by their initials in Table 1.

Next I asked the second student if she was acquainted with the other students with the exception of the first student; I assumed that if the first student said she was acquainted with the second, then the second was acquainted with the first. In other words, I assumed that the relationship of being acquainted with someone is *symmetric*. As a consequence, I only had to ask questions to fill in the upper half of Table 1; the lower half of the table was obtained by mirror reflection. I also assumed that a student is always acquainted with his or her self, which is represented in the table by all 1s along the diagonal from the top left to the bottom right.

2 Data Analysis from the Table

We can see from the table that there is just one student, CS, who is not acquainted with anyone else in the class (other than herself). It is a little harder to see that there are three students, RG, ST, and SP, who are acquainted with only one student in the class (other than themselves). Otherwise, it is rather difficult to interpret the table.

3 Data Analysis from the Graph

After completing the table, I then used a program called `circo` from the `GraphViz` package to represent the data in the form of a *graph* (see Figure 1. The program runs automatically on the data (after it was massaged slightly

with a short `perl` program), but `circo` would always choose to place the CS node inside the circle, so that was corrected by manually editing the `circo` output file. Nodes on the graph represent students in the class (on January 25, 2006), and edges on the graph indicate that two students are acquainted. The absence of an edge indicates that two students are not acquainted. The loops connected to each node represent the fact that a student is assumed to be acquainted with his or her self.

It is much easier to comprehend the pattern of acquaintences in the class from the graph than it is from the matrix. You can see at a glance that there is one student who is not connected to any of the others, and that there are three students who are connected to just one other.

4 Degrees of Separation

We would like to figure out 'degrees of separation' from this data. In other words, what is the least number of steps that will take us from any node on the graph to any other node. Strictly speaking, the number of degrees of separation in the graph is 'infinity' because CS can never be connected to anyone else. However, separating the class into two subclasses, one containing just CS and one containing everyone else, we can expect that there is a finite number of degrees of separation in each subclass. The CS component of the graph is connected with just one degree of separation (or maybe zero degrees of separation; we ought to think about what that means). It's not immediately clear how many degrees of separation there are in the other component of Figure 1.

From the graph, it seems that if we try to connect the outlying nodes SP and ST, it will take us six moves. So we conjecture that the graph has six degrees of separation. However, we can't be sure on the basis of just checking that one case, so we seek a more reliable way of determining the number of degrees of separation in the graph.

5 Matrix Multiplication

We do so by using the matrix multiplication method discussed in the lectures. I wrote a short `perl` program to load the table into a matrix M and then multiply the matrix by itself. The result is the matrix M^2, which I placed in a new table, Table 2. The entries of this table are the number of ways of connecting one student to another in two steps. The numbers for CS are still 0, of course, but ignoring that we have fewer zeros in the new table.

Still, it is difficult to interpret the table, so I piped the information from the table back into `circo` to obtain a graphical representation, Figure 2. Two nodes on the second graph are connected with an edge if the pair of students in question can be connected by two steps. Since everyone (in the non-CS component) can be connected to at least two other students in two steps, `circo` has pulled the figure into a circle. It is clear either from missing edges in the graph or 0s in the table that not everyone is connected to everyone else in two steps.

I continued the process, forming the matrices M^3 (number of ways of connecting two students in three steps), M^4, etc., up to M^6, and then producing a graph for each of the tables.

6 Six Degrees of Separation

Following the progression of the graphs from Figure 2 to Figure 6, you can see that the graphs become more and more connected. They are also nicely patterned. However, if you have a good eye, you can see that Figure 5 is not completely connected. If you don't have a good eye, you could easily miss the subtle difference between Figures 5 and 6. It is more reliable to use the tables to determine whether the graph is completely connected or not. Ignoring the CS row and column, we see that Table 5 still has some zero entries, but Table 6 has no zero entries. Therefore anyone in the subclass without CS can be connected to anyone else in at most six steps.

7 Conclusion

We have found a reliable method of determining the number of degrees of separation in a group of people, and a nice method of representing the data visually.

Some obvious questions remain, however: what would be the effect of adding the students who weren't present? What if we ask our question of other groups or expand the group under consideration from the MATH 127 class to the class of all students at the university, etc.? Is it really possible to connect any two people in the world in six steps?

	RA	SB	CC	SD	AD	BE	RG	CH	AI	RM	SP	AR	JS	CS	KS	KT	ST	KAW	KMW	NW
RA	1	1	1	1	0	1	0	1	0	0	0	0	0	0	0	1	0	0	1	1
SB	1	1	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	1
CC	1	1	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	1
SD	1	1	1	1	0	0	0	1	0	1	0	0	0	0	0	0	0	1	1	1
AD	0	0	0	0	1	1	0	0	0	0	0	1	0	0	0	0	1	0	0	0
BE	1	0	0	0	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0
RG	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0
CH	1	1	1	1	0	0	0	1	1	0	0	0	1	0	1	1	0	0	0	0
AI	0	0	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	1	0	0
RM	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0
SP	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0
AR	0	0	0	0	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0
JS	0	0	0	0	0	0	1	1	1	0	0	0	1	0	0	0	0	0	0	0
CS	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
KS	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1	0	0	0	1
KT	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1	0	0	1	1
ST	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
KAW	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0
KMW	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1
NW	1	1	1	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1

Table 1: Adjacency matrix, one degree of separation

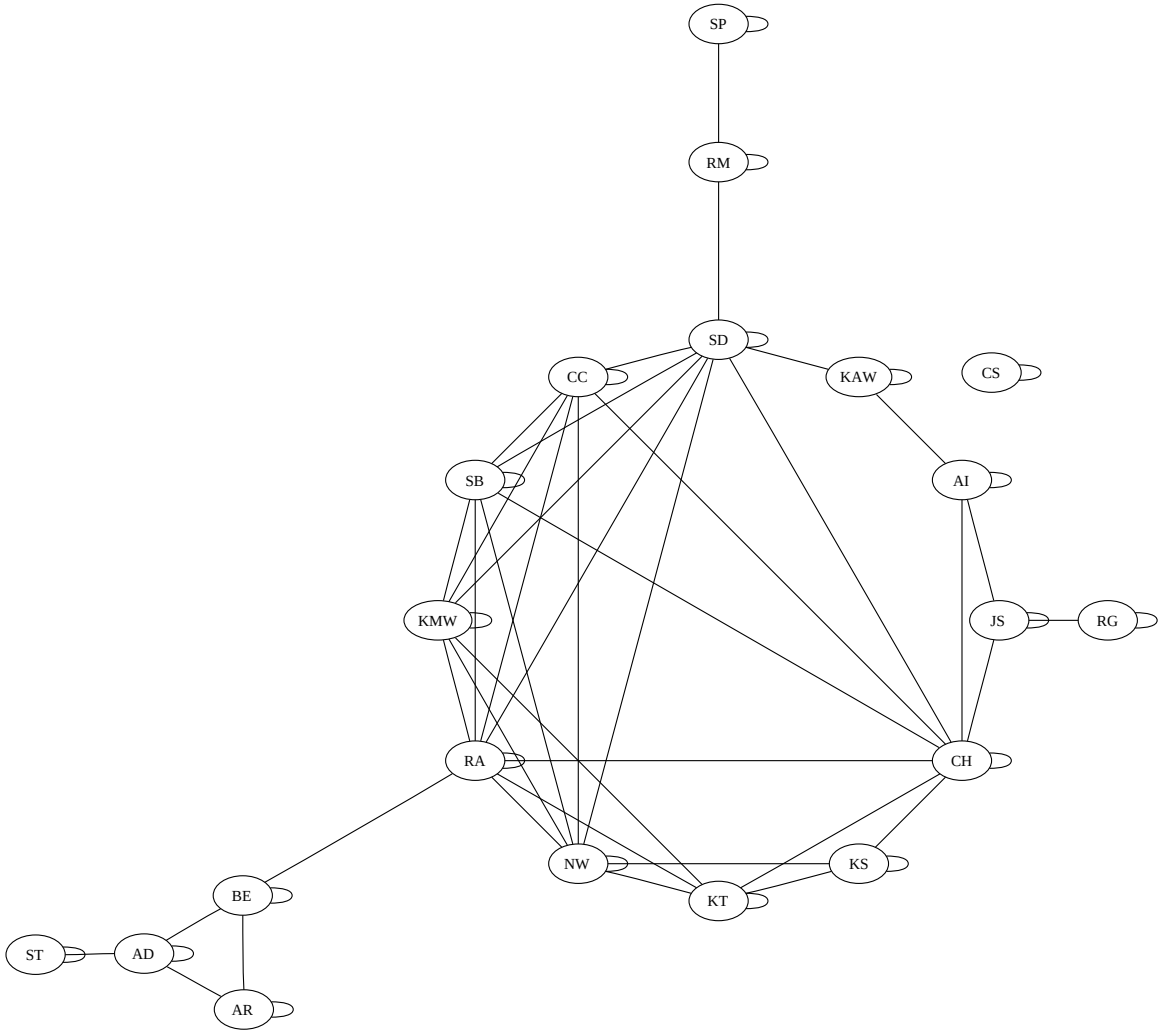

Figure 1: Class members connected by one degree of separation

	RA	SB	CC	SD	AD	BE	RG	CH	AI	RM	SP	AR	JS	CS	KS	KT	ST	KAW	KMW	NW
RA	9	7	7	7	1	2	0	6	1	1	0	1	1	0	3	5	0	1	7	7
SB	7	7	7	7	0	1	0	5	1	1	0	0	1	0	2	4	0	1	6	6
CC	7	7	7	7	0	1	0	5	1	1	0	0	1	0	2	4	0	1	6	6
SD	7	7	7	9	0	1	0	5	2	2	1	0	1	0	2	4	0	2	6	6
AD	1	0	0	0	4	3	0	0	0	0	0	3	0	0	0	0	2	0	0	0
BE	2	1	1	1	3	4	0	1	0	0	0	3	0	0	0	1	1	0	1	1
RG	0	0	0	0	0	0	2	1	1	0	0	0	2	0	0	0	0	0	0	0
CH	6	5	5	5	0	1	1	9	3	1	0	0	3	0	3	4	0	2	5	6
AI	1	1	1	2	0	0	1	3	4	0	0	0	3	0	1	1	0	2	0	0
RM	1	1	1	2	0	0	0	1	0	3	2	0	0	0	0	0	0	1	1	1
SP	0	0	0	1	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0
AR	1	0	0	0	3	3	0	0	0	0	0	3	0	0	0	0	1	0	0	0
JS	1	1	1	1	0	0	2	3	3	0	0	0	4	0	1	1	0	1	0	0
CS	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
KS	3	2	2	2	0	0	0	3	1	0	0	0	1	0	4	4	0	0	2	3
KT	5	4	4	4	0	1	0	4	1	0	0	0	1	0	4	6	0	0	4	5
ST	0	0	0	0	2	1	0	0	0	0	0	1	0	0	0	0	2	0	0	0
KAW	1	1	1	2	0	0	0	2	2	1	0	0	1	0	0	0	0	3	1	1
KMW	7	6	6	6	0	1	0	5	0	1	0	0	0	0	2	4	0	1	7	7
NW	7	6	6	6	0	1	0	6	0	1	0	0	0	0	3	5	0	1	7	8

Table 2: Adjacency matrix, two degrees of separation

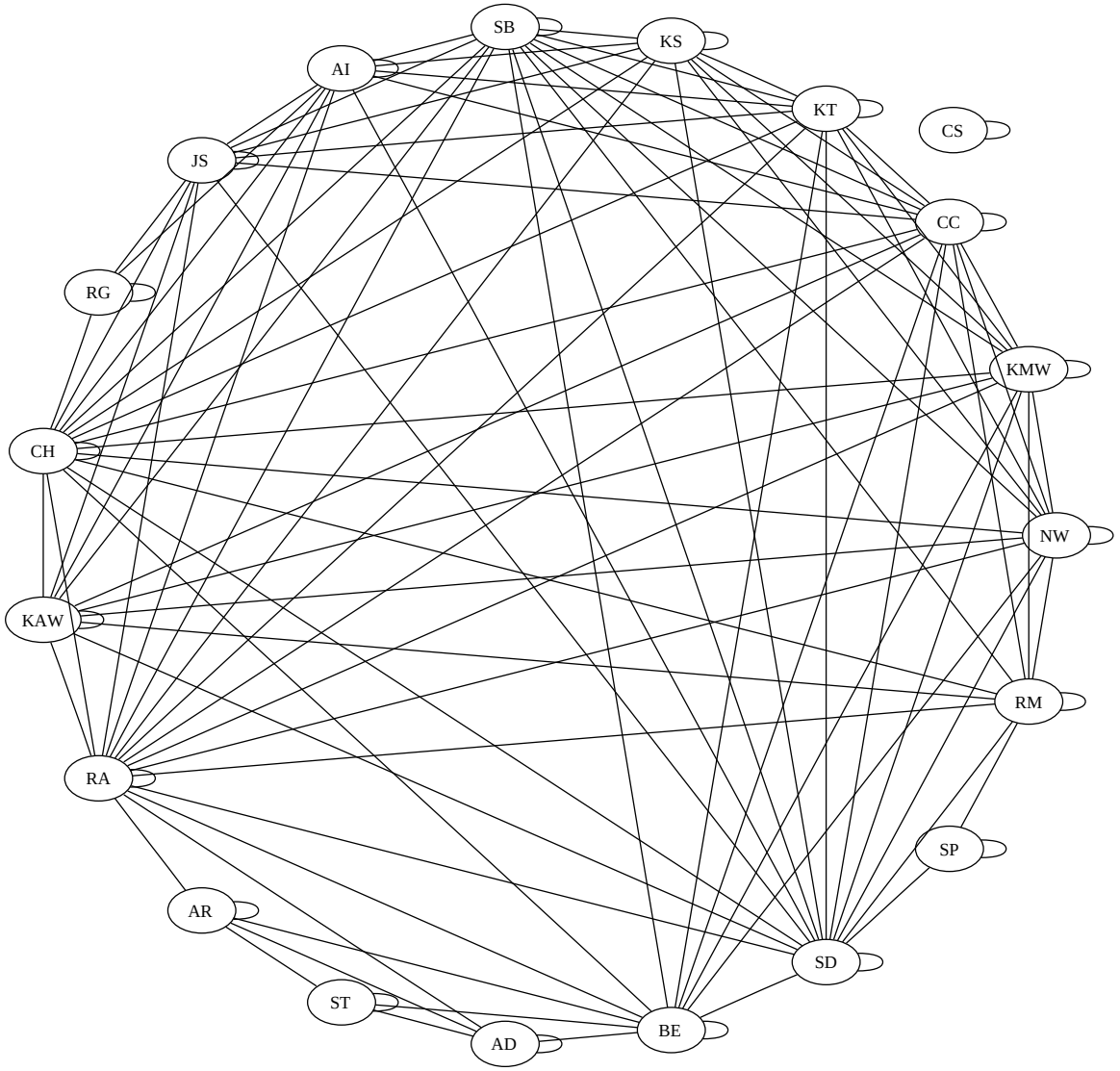

Figure 2: Class members connected by two degrees of separation

	RA	SB	CC	SD	AD	BE	RG	CH	AI	RM	SP	AR	JS	CS	KS	KT	ST	KAW	KMW	NW
RA	57	50	50	52	4	13	1	46	9	8	1	4	8	0	21	37	1	9	49	52
SB	50	45	45	47	1	8	1	41	8	8	1	1	7	0	17	30	0	9	44	46
CC	50	45	45	47	1	8	1	41	8	8	1	1	7	0	17	30	0	9	44	46
SD	52	47	47	51	1	8	1	44	10	12	3	1	8	0	17	30	0	13	46	48
AD	4	1	1	1	12	11	0	1	0	0	0	10	0	0	0	1	6	0	1	1
BE	13	8	8	8	11	12	0	7	1	1	0	10	1	0	3	6	4	1	8	8
RG	1	1	1	1	0	0	4	4	4	0	0	0	6	0	1	1	0	1	0	0
CH	46	41	41	44	1	7	4	43	17	6	1	1	16	0	22	33	0	10	36	39
AI	9	8	8	10	0	1	4	17	12	2	0	0	11	0	5	6	0	8	6	7
RM	8	8	8	12	0	1	0	6	2	7	5	0	1	0	2	4	0	3	7	7
SP	1	1	1	3	0	0	0	1	0	5	4	0	0	0	0	0	0	1	1	1
AR	4	1	1	1	10	10	0	1	0	0	0	9	0	0	0	1	4	0	1	1
JS	8	7	7	8	0	1	6	16	11	1	0	0	12	0	5	6	0	5	5	6
CS	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
KS	21	17	17	17	0	3	1	22	5	2	0	0	5	0	14	19	0	3	18	22
KT	37	30	30	30	1	6	1	33	6	4	0	1	6	0	19	28	0	5	32	36
ST	1	0	0	0	6	4	0	0	0	0	0	4	0	0	0	0	4	0	0	0
KAW	9	9	9	13	0	1	1	10	8	3	1	0	5	0	3	5	0	7	7	7
KMW	49	44	44	46	1	8	0	36	6	7	1	1	5	0	18	32	0	7	43	45
NW	52	46	46	48	1	8	0	39	7	7	1	1	6	0	22	36	0	7	45	48

Table 3: Adjacency matrix, three degrees of separation

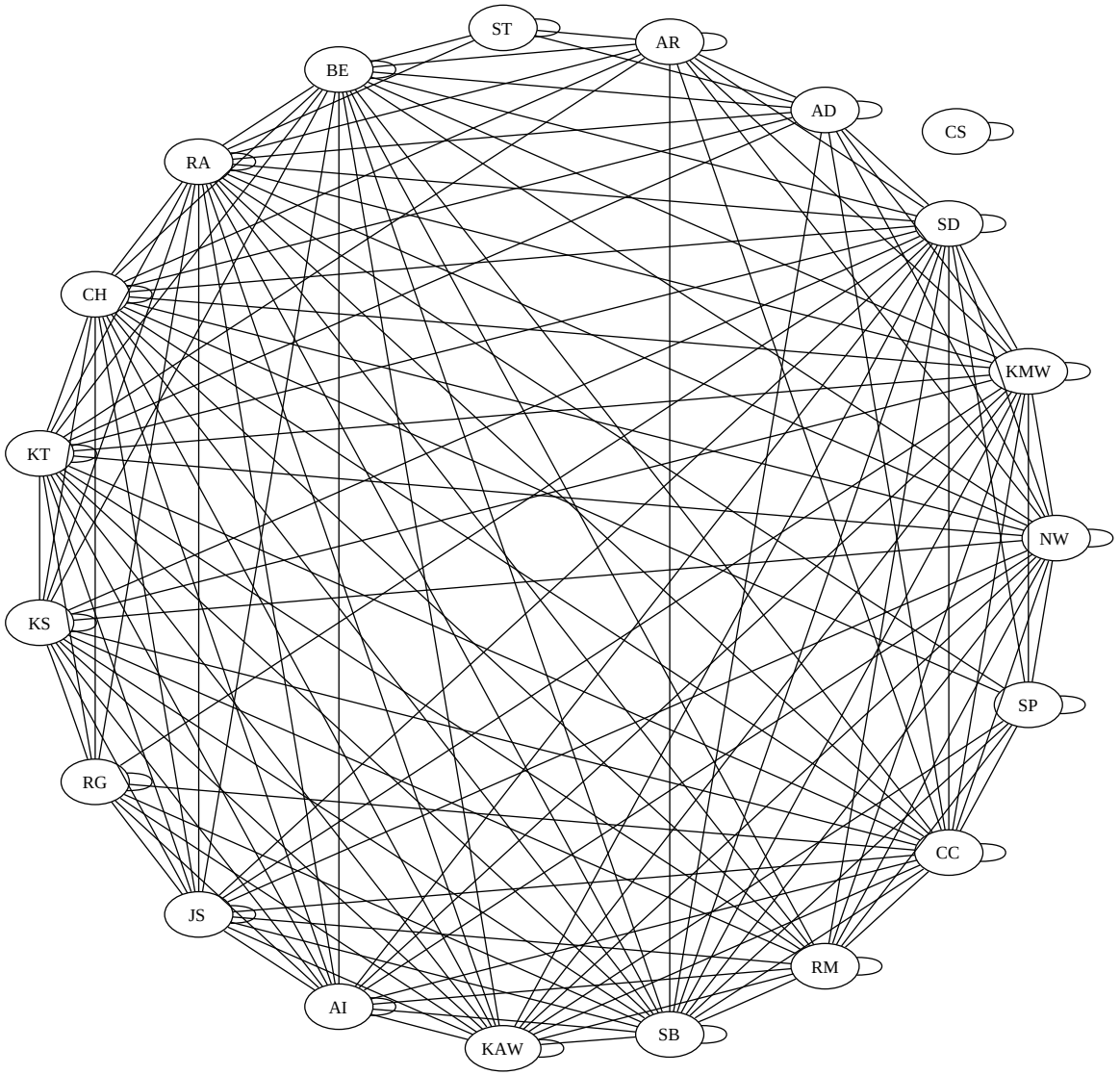

Figure 3: Class members connected by three degrees of separation

	RA	SB	CC	SD	AD	BE	RG	CH	AI	RM	SP	AR	JS	CS	KS	KT	ST	KAW	KMW	NW
RA	406	356	356	373	22	78	9	330	72	61	9	21	64	0	156	262	5	70	347	368
SB	356	318	318	335	10	60	8	290	65	56	9	10	57	0	134	228	1	64	307	324
CC	356	318	318	335	10	60	8	290	65	56	9	10	57	0	134	228	1	64	307	324
SD	373	335	335	360	10	62	9	306	75	66	15	10	63	0	139	237	1	74	321	338
AD	22	10	10	10	39	37	0	9	1	1	0	33	1	0	3	8	18	1	10	10
BE	78	60	60	62	37	46	1	55	10	9	1	33	9	0	24	45	15	10	59	62
RG	9	8	8	9	0	1	10	20	15	1	0	0	18	0	6	7	0	6	5	6
CH	330	290	290	306	9	55	20	303	86	51	7	9	80	0	137	219	1	71	280	302
AI	72	65	65	75	1	10	15	86	48	12	2	1	44	0	35	50	0	30	54	59
RM	61	56	56	66	1	9	1	51	12	24	12	1	9	0	19	34	0	17	54	56
SP	9	9	9	15	0	1	0	7	2	12	9	0	1	0	2	4	0	4	8	8
AR	21	10	10	10	33	33	0	9	1	1	0	29	1	0	3	8	14	1	10	10
JS	64	57	57	63	1	9	18	80	44	9	1	1	45	0	33	46	0	24	47	52
CS	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
KS	156	134	134	139	3	24	6	137	35	19	2	3	33	0	77	116	0	25	131	145
KT	262	228	228	237	8	45	7	219	50	34	4	8	46	0	116	185	1	41	223	242
ST	5	1	1	1	18	15	0	1	0	0	0	14	0	0	0	1	10	0	1	1
KAW	70	64	64	74	1	10	6	71	30	17	4	1	24	0	25	41	0	28	59	62
KMW	347	307	307	321	10	59	5	280	54	54	8	10	47	0	131	223	1	59	303	321
NW	368	324	324	338	10	62	6	302	59	56	8	10	52	0	145	242	1	62	321	343

Table 4: Adjacency matrix, four degrees of separation

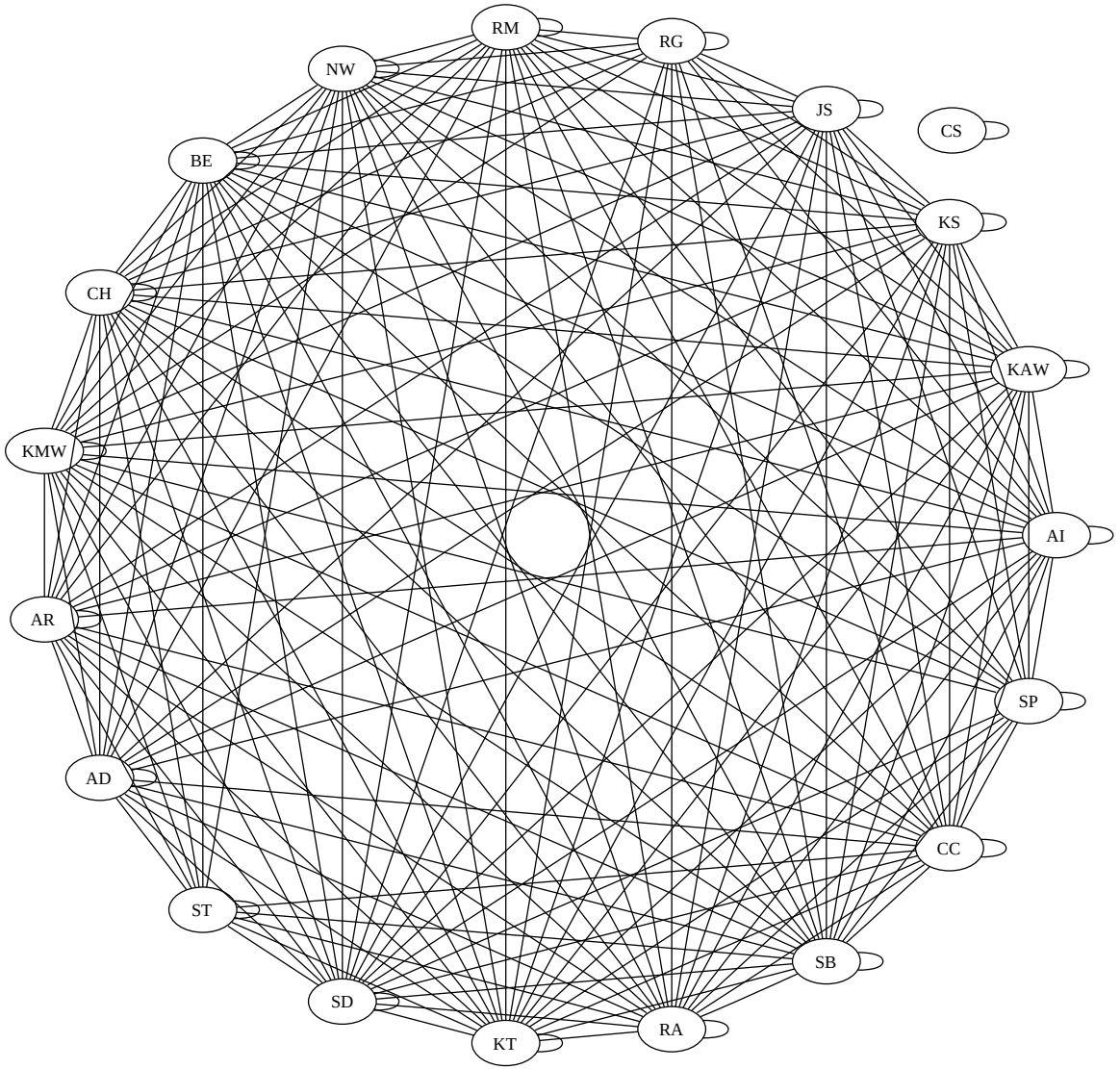

Figure 4: Class members connected by four degrees of separation

	RA	SB	CC	SD	AD	BE	RG	CH	AI	RM	SP	AR	JS	CS	KS	KT	ST	KAW	KMW	NW
RA	2876	2536	2536	2667	126	527	73	2375	536	443	70	121	475	0	1116	1869	27	515	2468	2624
SB	2536	2248	2248	2368	81	436	65	2101	476	400	65	80	420	0	976	1639	11	464	2186	2320
CC	2536	2248	2248	2368	81	436	65	2101	476	400	65	80	420	0	976	1639	11	464	2186	2320
SD	2667	2368	2368	2508	83	455	72	2223	518	441	81	82	453	0	1020	1714	11	509	2299	2438
AD	126	81	81	83	127	131	1	74	12	11	1	109	11	0	30	62	57	12	80	83
BE	527	436	436	455	131	194	10	403	84	72	10	116	75	0	186	323	52	82	426	450
RG	73	65	65	72	1	10	28	100	59	10	1	1	63	0	39	53	0	30	52	58
CH	2375	2101	2101	2223	74	403	100	2041	540	364	58	73	489	0	961	1571	10	463	2017	2154
AI	536	476	476	518	12	84	59	540	208	89	14	12	193	0	230	356	1	153	440	475
RM	443	400	400	441	11	72	10	364	89	102	36	11	73	0	160	275	1	95	383	402
SP	70	65	65	81	1	10	1	58	14	36	21	1	10	0	21	38	0	21	62	64
AR	121	80	80	82	109	116	1	73	12	11	1	95	11	0	30	61	47	12	79	82
JS	475	420	420	453	11	75	63	489	193	73	10	11	187	0	211	322	1	131	386	419
CS	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
KS	1116	976	976	1020	30	186	39	961	230	160	21	30	211	0	475	762	3	199	955	1032
KT	1869	1639	1639	1714	62	323	53	1571	356	275	38	61	322	0	762	1247	9	328	1605	1721
ST	27	11	11	11	57	52	0	10	1	1	0	47	1	0	3	9	28	1	11	11
KAW	515	464	464	509	12	82	30	463	153	95	21	12	131	0	199	328	1	132	434	459
KMW	2468	2186	2186	2299	80	426	52	2017	440	383	62	79	386	0	955	1605	11	434	2129	2260
NW	2624	2320	2320	2438	83	450	58	2154	475	402	64	82	419	0	1032	1721	11	459	2260	2405

Table 5: Adjacency matrix, five degrees of separation

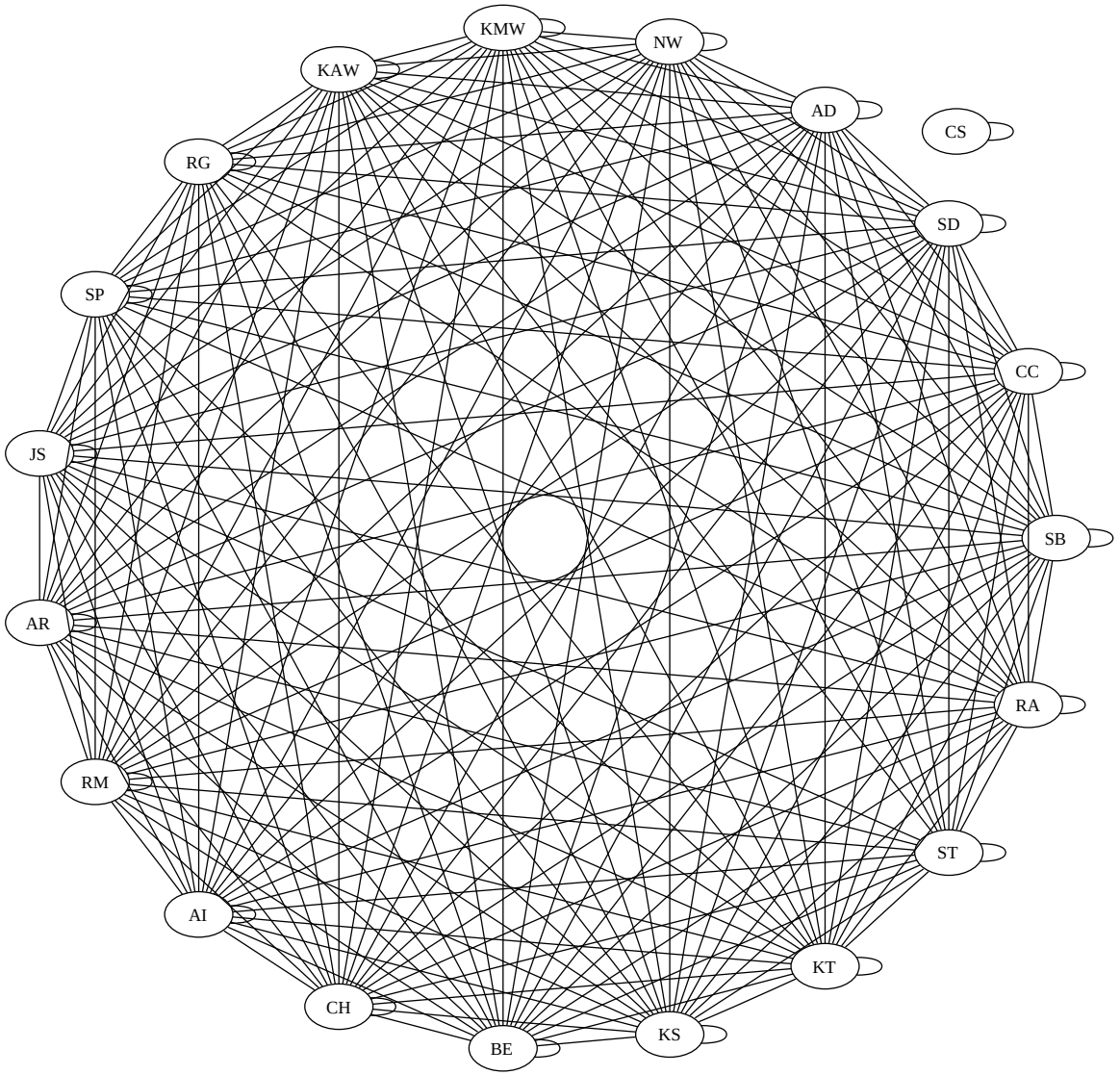

Figure 5: Class members connected by five degrees of separation

	RA	SB	CC	SD	AD	BE	RG	CH	AI	RM	SP	AR	JS	CS	KS	KT	ST	KAW	KMW	NW
RA	20478	18082	18082	19040	801	3650	548	16986	3901	3180	513	774	3459	0	7984	13328	153	3718	17576	18692
SB	18082	16007	16007	16871	608	3133	485	15012	3461	2833	465	597	3062	0	7036	11758	92	3308	15545	16521
CC	18082	16007	16007	16871	608	3133	485	15012	3461	2833	465	597	3062	0	7036	11758	92	3308	15545	16521
SD	19040	16871	16871	17821	631	3287	525	15839	3703	3030	522	620	3266	0	7395	12361	94	3535	16362	17382
AD	801	608	608	631	424	493	12	560	109	95	12	367	98	0	249	455	184	107	596	626
BE	3650	3133	3133	3287	493	968	85	2925	644	537	82	441	572	0	1362	2315	183	621	3053	3239
RG	548	485	485	525	12	85	91	589	252	83	11	12	250	0	250	375	1	161	438	477
CH	16986	15012	15012	15839	560	2925	589	14402	3533	2645	422	550	3170	0	6727	11119	84	3226	14542	15503
AI	3901	3461	3461	3703	109	644	252	3533	1094	621	103	108	1000	0	1601	2577	13	879	3277	3507
RM	3180	2833	2833	3030	95	537	83	2645	621	579	138	94	536	0	1201	2027	12	625	2744	2904
SP	513	465	465	522	12	82	11	422	103	138	57	12	83	0	181	313	1	116	445	466
AR	774	597	597	620	367	441	12	550	108	94	12	320	97	0	246	446	156	106	585	615
JS	3459	3062	3062	3266	98	572	250	3170	1000	536	83	97	932	0	1441	2302	12	777	2895	3106
CS	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
KS	7984	7036	7036	7395	249	1362	250	6727	1601	1201	181	246	1441	0	3230	5301	33	1449	6837	7312
KT	13328	11758	11758	12361	455	2315	375	11119	2577	2027	313	446	2302	0	5301	8775	71	2398	11434	12196
ST	153	92	92	94	184	183	1	84	13	12	1	156	12	0	33	71	85	13	91	94
KAW	3718	3308	3308	3535	107	621	161	3226	879	625	116	106	777	0	1449	2398	13	794	3173	3372
KMW	17576	15545	15545	16362	596	3053	438	14542	3277	2744	445	585	2895	0	6837	11434	91	3173	15133	16088
NW	18692	16521	16521	17382	626	3239	477	15503	3507	2904	466	615	3106	0	7312	12196	94	3372	16088	17120

Table 6: Adjacency matrix, six degrees of separation

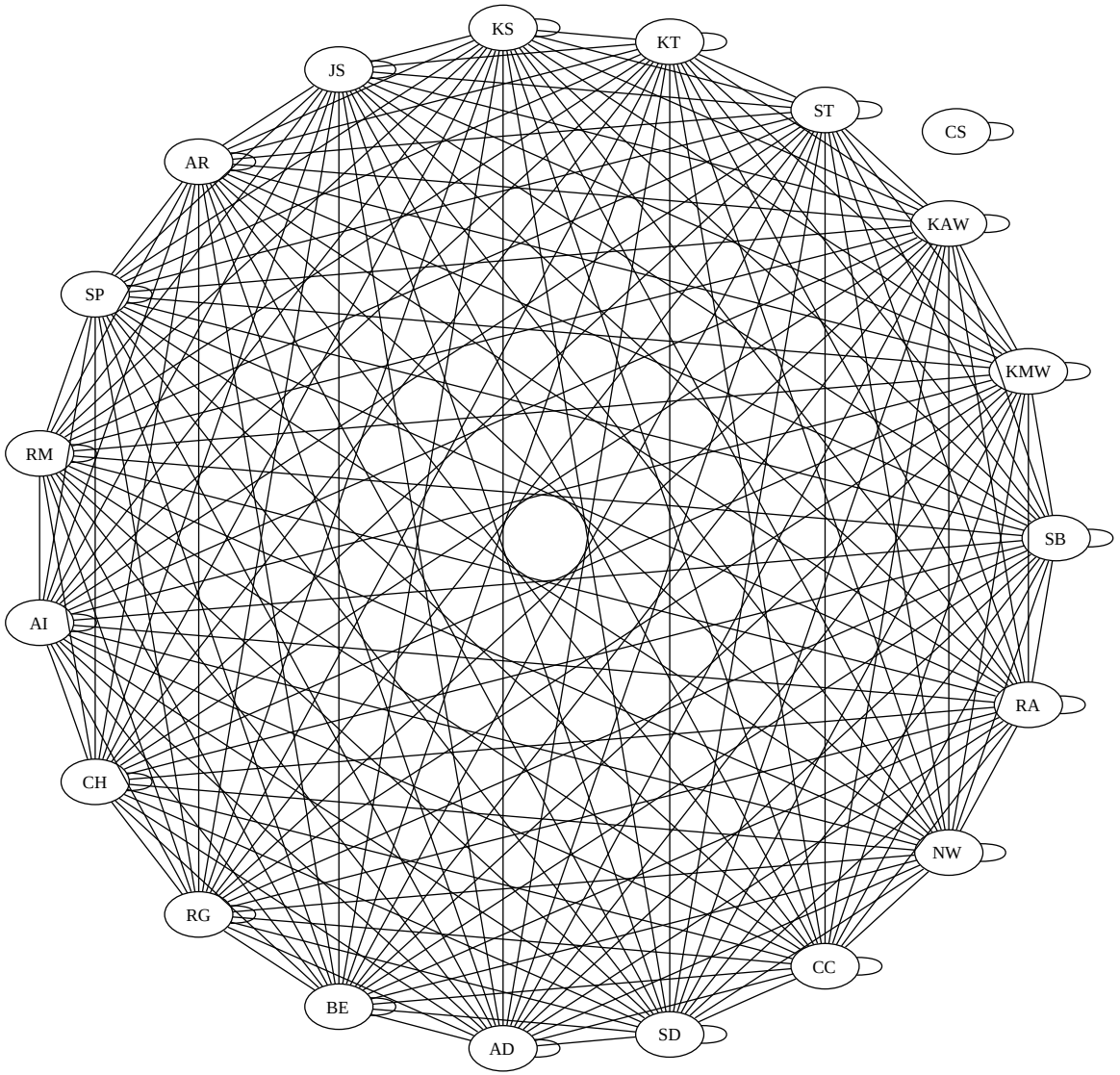

Figure 6: Class members connected by six degrees of separation