

# MATH221-001 200530 Sample Term Test 1

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Please answer each of the following questions. A non-programmable calculator is allowed. The test is worth a total of 45 marks; you should be able to earn about 1 mark per minute, which will give you 5 minutes to check your work. The last two problems are a little harder than the others, and are meant to distinguish A and B level students from C level students.

1. (2 marks) Find numbers  $q$  and  $r$  such that  $1860 = 48q + r$ ,  $0 \leq r < 48$ .
2. (3 marks) Find the greatest common divisor of 1860 and 48.
3. (5 marks) Find integers  $m$  and  $n$  such that  $\gcd(1860, 48) = 1860m + 48n$ .
4. (4 marks) Find a pair of integers  $p$  and  $q$ , different from the pair  $m$  and  $n$  found in the previous problem, such that  $\gcd(1860, 48) = 1860p + 48q$ .
5. (3 marks) Prove that the number 173 is prime. (Hint: you only have to check that no prime less than or equal to the square root of 173 divides 173. You can use a calculator if you wish.)
6. (5 marks) Find the prime factorization of 266420.
7. (5 marks) Given any integers  $a$  and  $b$  (not both zero), let  $d = \gcd(a, b)$ . Show that if  $D$  is any multiple of  $d$ , then there are integers  $m$  and  $n$  such that  $D = ma + nb$ .
8. (8 marks) Given integers  $a$  and  $b$ ,  $b > 0$ , prove that there are unique numbers  $q$  and  $r'$  such that  $a = bq + r'$ ,  $-b/2 < r' \leq b/2$ .
9. (5 marks) Given any two integers  $a$  and  $b$ , one even and one odd, show that  $\gcd(a, b) = \gcd(a+b, a-b)$ .
10. (5 marks) The first few Fibonacci numbers are  $f_1 = 1$ ,  $f_2 = 1$ ,  $f_3 = 2$ ,  $f_4 = 3$ ,  $f_5 = 5$ ,  $f_6 = 8$ ,  $f_7 = 13$ ,  $f_8 = 21$ , and so on, where  $f_{n+2} = f_{n+1} + f_n$  for  $n \geq 1$ . The result

$$\gcd(f_m, f_n) = f_{\gcd(m,n)} \tag{1}$$

was established in Problem Set 1. Use that result to show that if  $m$  divides  $n$  then  $f_m$  divides  $f_n$ . Verify the result for a few values of  $m$  and  $n$  both greater than 2.