MATH221-001 200530 Term Test 1

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Please answer each of the following questions. A non-programmable calculator is allowed. The test is worth a total of 45 marks; you should be able to earn about 1 mark per minute, which will give you 5 minutes to check your work. The last two problems are a little harder than the others, and are meant to distinguish A and B level work from C level work.

1. (2 marks) Find numbers q and r such that $378 = 322q + r$, $0 \leq r < 322$.

2. (3 marks) Find the greatest common divisor of 378 and 322.

3. (5 marks) Find integers m and n such that $378m + 322n = 28$.

4. (4 marks)

   (a) Find a pair of integers p and q, different from the pair m and n found in problem 3, such that $378p + 322q = 28$.

   (b) Find a third pair of integers r and s, different from the pair m and n found in problem 3 and different from the pair p and q found in problem 4a, such that $378r + 322s = 28$.

5. (3 marks) Prove that the number 391 is composite. (You can use a calculator if you wish.)

6. (5 marks) Find the prime factorization of 766360.

7. (5 marks) Given any two integers a and b, not both 0, let $d = \gcd(a, b)$. Prove that the linear combination $ma + nb$ is a multiple of d for any integers m and n.

8. (8 marks) (Division with 'least non-positive' remainder.) Given integers a and b, $b > 0$, prove that there are unique numbers q and r such that $a = bq + r$ with $-b < r \leq 0$. (For example, dividing 20 by 3 in this way gives $20 = 3 \times 7 - 1$ instead of $20 = 3 \times 6 + 2$.)

9. (5 marks) Prove that if a, b, c are any three integers satisfying $\gcd(a, b) = 1$ and $\gcd(a, c) = 1$, then $\gcd(a, bc) = 1$.

10. (5 marks) The first few Fibonacci numbers are $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$, $f_5 = 5$, $f_6 = 8$, $f_7 = 13$, $f_8 = 21$, and so on, where $f_{n+2} = f_{n+1} + f_n$ for $n \geq 1$. Show that any positive integer can be written as a sum of distinct Fibonacci numbers. (For example, we have $6 = 3 + 2 + 1$, but we can't write $6 = 3 + 3$ because 3 appears twice; $6 = 2 + 2 + 2$ and $6 = 1 + 1 + 1 + 1 + 1 + 1$ are also not allowed.)