

MATH221-001 200530 Practice Midterm Test 2 DRAFT

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Please answer each of the following questions. A non-programmable calculator is allowed. The test is worth a total of 45 marks; you should be able to earn about 1 mark per minute, which will give you 5 minutes to check your work. The last two problems are a little harder than the others, and are meant to distinguish A and B level work from C level work.

- (5 marks) Fill in the following truth table and use it to argue that $\neg(p \wedge q) \equiv (\neg p) \vee (\neg q)$. [TODO: partial truth table goes here.]
- (5 marks) Suppose that A and B are subsets of some universal set X , and define A' to be the set of all elements of X not in A . Use the previous result to show that $(A \cap B)' = A' \cup B'$.
- (5 marks) Consider the statement $p =$ “if n is not a multiple of 7 then n is a multiple of 3”. Find statements q and r such that $p = q \Rightarrow r$, and then write down in symbols and in words the contrapositive of p , the converse of p , and the contrary of p .
- (5 marks) Using the only the axioms [TODO: provide axioms], prove that

$$(b + c) \times a = (b \times a) + (c \times a)$$

for all natural numbers a , b , and c .

- Consider the statement $P(n) =$ “ $n^3 + 5n$ is a multiple of 6”. Show that $P(n)$ is true for all $n \in \mathbb{N}$ by following the steps below.
 - (2 marks) Identify and prove the base case.
 - (1 mark) Identify the induction hypothesis.
 - (5 marks) Identify and prove the induction step

(Hint: you may use the identity $(k + 1)^3 = k^3 + 3k^2 + 3k + 1$.)

- (8 marks) Prove that $\sum_{i=1}^n i(i + 1) = \frac{n(n + 1)(n + 2)}{3}$ for all $n \in \mathbb{N}$.

- Suppose you have an unlimited number of 4-cent and 9-cent stamps.
 - (2 marks) Make 39 cents in postage with those stamps.
 - (5 marks) Show that it is possible to make any value n in postage where $n \geq n_0$ where $n_0 = 36$.
 - (2 marks) What is the smallest value of n_0 for which the above statement is true?