MATH221-001 200530 Midterm Test 2

Edward Doolittle

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Please answer each of the following questions. A non-programmable calculator is allowed. The test is worth a total of 45 marks; you should be able to earn about 1 mark per minute, which will give you 5 minutes to check your work. The last problem is a little harder than the others, and is meant to distinguish A and B level work from C level work.

1. (5 marks) Fill in the following truth table and provide an argument to show that the logical expressions $\neg(p \wedge \neg q)$ and $\neg p \vee q$ are logically equivalent.

p	q	p	$\neg q$	$p \wedge \neg q$	$\neg(p \wedge \neg q)$	$\neg p$	q	$\neg p \vee q$
F	F	F				T		
F	T	F				T		
T	F	T	T			F	F	
T	T	T	F			F	T	

2. (5 marks) Suppose that A and B are subsets of some universal set X, and define A' to be the set of all elements of X not in A. Show that $(A \cap B')' = A' \cup B$.

3. (6 marks) Consider the statement $p =$ "if m^2 is a multiple of 2 then m is a multiple of 2". Find statements q and r such that $p = (q \Rightarrow r)$, and then write down in symbols and in words the contrapositive of p. Is p true? Prove or give a counter-example.

4. (5 marks) Using the only the axioms on the second page, prove that
$$(x+y)^2 = x^2 + 2xy + y^2$$
for all natural numbers x and y.

5. Consider the statement $P(n) =$ "$3^{2n+1} + 1$ is a multiple of 4". Show that $P(n)$ is true for all $n \in \mathbb{N}$ by following the steps below.
   (a) (2 marks) Identify and prove the base case.
   (b) (1 mark) Identify the induction hypothesis.
   (c) (5 marks) Identify and prove the induction step

6. (8 marks) Prove that $\sum_{i=1}^{n} i(i+2) = \frac{n(n+1)(2n+7)}{6}$ for all $n \in \mathbb{N}$.

7. (8 marks) Prove that every third Fibonacci number f_{3n} is even. (Recall $f_1 = 1$, $f_2 = 1$, and $f_{n+2} = f_n + f_{n+1}$ for $n \in \mathbb{N}$.)