MATH 221-001 200530 Problem Set 2

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Due: Friday, September 30, 2005

1. Without using a calculator or carrying out any "long multiplication", show that
   a. $3726353 \times 1939678 \equiv 4 \bmod 10$
   b. $4939883 \times 385677 \equiv -9 \bmod 25$
2. Use the technique of "casting out nines" to determine that one of the following statements is untrue. Is the other statement also untrue? (Hint: You might want to apply casting out nines multiple times, e.g., $9837233 \equiv 9 + 8 + 3 + 7 + 2 + 3 + 3 = 35 \equiv 3 + 5 = 8 \bmod 9$ is obtained by "casting out nines" twice.)
   a. $3726353 \times 1939678 = 7227924943334$
   b. $4939883 \times 385677 = 1905199255691$
3. Explain how the web site http://www.mysticalball.com/ works.
4. Express the fact that if 40,000,000 is a multiple of 375 then so is 120,000,000 in the theorem-proof style presented in chapter 1 of the textbook. Is 40,000,000 a multiple of 375? Is that fact relevant to your answer to the first part of this question?
5. Find the least positive residues of the numbers 329, 666, and 291 mod 11. Also, for each of those numbers, add the two outer digits and subtract the middle digit. How are the results related? State and prove a general result that applies to all numbers, not just three digit numbers. (See the problems in the textbook for a hint.)
6. Explain how the web site http://www.digicc.com/fido/ might work. Why does it ask you not to circle a zero?
7. Use the day of the week formula presented in the lectures to determine the date of Labour Day (first Monday in September) in the year 1973 without using a calendar. (Bonus question: give another example of a holiday for which the formula can help you determine the date of the holiday given the year, and an example of a holiday for which the formula does not help determine the date of the holiday given the year.)
8. Find the error in the following argument: Let a and b be any two distinct (unequal) numbers. Let c be their average, so that $a + b = 2c$. Multiply both sides of the equation by $a-b$ to obtain $a^2 - b^2 = 2ac - 2bc$. Add $b^2 - 2ac + c^2$ to both sides to obtain $a^2 - 2ac + c^2 = b^2 - 2bc + c^2$. Both sides of the previous equation are now perfect squares and the equation can be written $(a-c)^2 = (b-c)^2$. Take the square root of both sides; then $a - c = b - c$, from which it follows that $a = b$, contrary to our initial assumption.
9. Note that $1^2 = 1$, $3^2 = 9$, and $5^2 = 25$ are all one more than a multiple of 8. Is it true that the square of any odd number is one more than a multiple of 8? Can you prove that your answer is correct?
10. Does the formula $2n^2 + 29$ generate prime numbers for every nonnegative value of n? Why or why not?