MATH221-001 200530 Problem Set 8 Hints

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1. This is a problem directly from the textbook; your solution can be modeled on the simpler problem solved in the textbook.

2. The number of three letter arrangements is ordered selection with repetition from letters of the alphabet; the number of three number arrangements is selection with repetition from digits. Multiply the two together to get the total number of license plates.

3. This is unordered selection with repetition; you can find the formula in one of the sections of the textbook we haven't yet covered.

4. This is ordered selection without repetition.

5. Hmm, I think there may be a pattern forming in the last three questions. Guess what this is.

6. The textbook provides hints for solving this both by a counting argument and by induction.

7. I would try to prove this identity using induction, although there are other approaches.

8. Find an appropriate alternate expression for $x^2+2xy+y^2$.

9. This is one of the most challenging problems I have assigned. It can be solved using a variation of the binomial theorem for fractional exponents: $(1+x)^{-1/2}$ can be expressed using an infinite binomial expansion. Since we haven't covered infinite series, you should try to find another way to solve the problem.

   Here are some other approaches:

   (a) An inebriated individual starts at $(0,0)$ and takes series of steps either northeast or southeast. The number of ways he can take $2n$ steps is 2^{2n}. Now classify his different possible walks by the number $2k$ which is the last time he crossed the x-axis.

   (b) Prove that $\binom{2n}{n} = \frac{4n-2}{n}\binom{2(n-1)}{n-1}$ and use that identity to try to get an induction going.

   (c) Try replacing the central binomial coefficient $\binom{2n}{n}$ with the expression $\sum_{k=0}^{n}\binom{n}{k}^2$ that we proved in the lectures.

   (d) The identity

   $$\sum_{k=0}^{n}\binom{2n}{2k}^2 = \frac{1}{2}\left(\binom{4n}{2n} + (-1)^n\binom{2n}{n}\right)$$

   may also help.

10. This is from the textbook; you may find some help in the section of the textbook from which the problem was taken.

    Consider the two diagrams of the subsets of $\mathbb{N}_2$ and $\mathbb{N}_3$ in Figure 1. In both cases, if we pick two subsets that can be connected by a vertical 'chain', then one set is a subset of another, and we are in trouble. So we must avoid chains; what we want is actually called an 'antichain'. Perhaps we can avoid chains by staying in just one

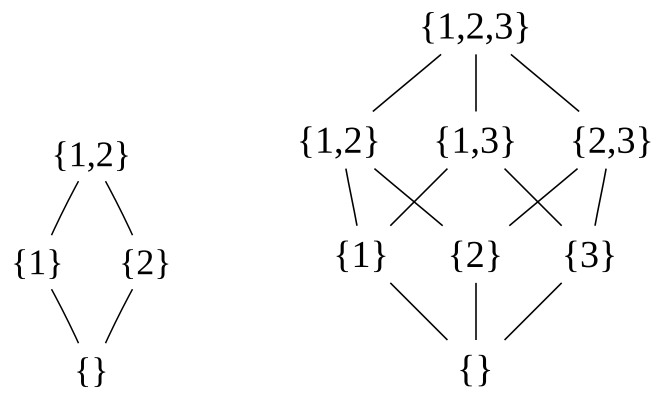

Figure 1: Subset Lattices for $\{1,2\}$ and $\{1,2,3\}$

level of the lattice? Try drawing similar diagrams for $\mathbb{N}_4$ and $\mathbb{N}_5$ and then generalizing. But remember, even if you get the picture right, there are still some things to prove.

How does Pascal's triangle fit into this?