MATH221-001 200530 Problem Set 8

Edward Doolittle

Due: Friday, December 2, 2005

1. Show that in any set of 10 people there are either four mutual friends or three mutual strangers. (A set of four people are mutual friends if any pair of people in the set are friends.)

2. A Saskitoba license plate consists of three letters followed by three digits. How many different Sasktoba license plates are there?

3. Suppose I am allowed to pick my own set of letters and numbers (but not their arrangement) for a Saskitoba licence plate of the type in question 2. In how many ways can I do so?

4. How many different Saskitoba license plates are there if any letter or number cannot be used twice in the same plate?

5. Suppose I am allowed to pick three different letters and three different numbers to make up my own Saskitoba licence plate of the type in question 4. In how many ways can I do so? Given a choice of letters and numbers for a Saskitoba licence plate in the previous problem, how many rearrangements are there of the given plate which still satisfy the requirements of Sasktoba plates (three letters followed by three numbers)?

6. Prove that $\binom{s-1}{0} + \binom{s}{1} + \cdots + \binom{s+n-2}{n-1} + \binom{s+n-1}{n} = \binom{s+n}{n}$ where $s, n \in \mathbb{N}$.

7. Show that when $n \geq m$, $\binom{m}{m} + \binom{m+1}{m} + \cdots + \binom{n}{m} = \binom{n+1}{m+1}$.

8. What is the coefficient of x^2y^4 in the expansion of $(x^2 + 2xy + y^2)^3$?

9. Show that $\sum_{k=0}^{n} \binom{2k}{k} \binom{2n-2k}{n-k} = 2^{2n}$.

10. As a partial answer to the question posed in solutions 6, item 7, show that there is a collection of $\binom{n}{n^*}$ subsets of $\mathbb{N}_n$ with the property that not one of them is contained in another. Here n^* means $n/2$ if n is even and $(n-1)/2$ if n is odd.