MATH221-001 200630 Group Work Assignment 1: The Pigeonhole Principle

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Please try to solve as many of the following problems as you can with your group. At the end, hand in one set of answers for everyone in the group. Make sure that everyone in your group puts their name on the answer pages. The problems will be marked, and everyone in the group will get the same grade. You problably won't have time do do all the problems, so select the ones you can do best. Save the geometry problems for last; they are quite hard.

1. **Socks.**
   (a) My sock drawer contains 3 different kinds of socks. How many socks do I have to take from the drawer to ensure that I have two socks of the same kind?
   (b) My other sock drawer contains 7 different kinds of socks. How many socks do I have to take from the drawer to ensure that I have two socks of the same kind?
   (c) My alien cousin has three feet. He has a sock drawer containing 5 different kinds of socks. How many socks must he pick from his drawer to be sure of having three socks of the same kind?
2. **Birthdays.**
   (a) How many people do I need to have in a room to ensure that two of them were born on the same day of the week?
   (b) How many people do I need to have in a room to ensure that two of them were born on the same day of the month. (A person born on February 6 and a person born on August 6 were "born on the same day of the month".)
   (c) How many people do I need to have in a room to ensure that two of them celebrate their birthday on the same day?
   (d) Show that (under reasonable assumptions) there must be five people in Regina who are exactly the same age (i.e., they were all born on the same day).
3. **Placing dominos on a chess board.** Suppose we have a supply of dominos of size 2×1 that will exactly cover 2 squares of our chessboard.
   (a) Show that we can't arrange dominos on a 3×3 chessboard to cover every square.
   (b) Show that we can arrange dominos on a 4×4 chessboard to cover every square.
   (c) Show that if we remove two adjacent corners from a 4×4 chessboard we can still cover it with dominos.
   (d) Show that it we remove two opposite corners from a 4×4 chessboard we cannot cover it with dominos.
4. Suppose I select 55 numbers from the set $\{1, 2, 3, \cdots, 100\}$.
   (a) Show that I must have two numbers which differ by 9.
   (b) Show that I must have two numbers which differ by 10.
   (c) Show that I must have two numbers which differ by 12.
   (d) Show that I must have two numbers which differ by 13.

(e) Show that I do not necessarily have to have two numbers which differ by 11.

5. **Lossless compression.** Suppose that we purport to have a file compression scheme that is "lossless". We model the compression scheme in the following way: a function C from the set of computer files to the set of computer files which is an injection (lossless) and for which a file of size m is always mapped to a file of size m or smaller. For us to call this a compression scheme, at least one file must be mapped to a smaller file.

   (a) Let M be the least number such that there is a file F of length M that compressing into something shorter; say $C(F) = G$ where the length of G is N, where N is less than M. Why does M have to exist?

   (b) Show that every file of length N keeps its size after compression.

   (c) Show that there are 2^N files of length N.

   (d) Show that there are $2^N + 1$ files which map into files of length N.

   (e) Show that C cannot be an injection.

   (f) What can we say about file compression schemes in general based on the above?

6. **Geometry. (hard ones)**

   (a) Prove that among any five points selected inside an equilateral triangle of side 1 unit, at least two of the points must have distance no greater than 0.5 units from one another.

   (b) Given any six points inside a circle of radius 1, show that some pair of points must have distance no greater than 1 unit from one another.

   (c) Suppose that every point in the plane is coloured red or blue. Show that there must be some rectangle with all four corners the same colour.

   (d) Suppose that every point on (the circumference of) a circle is coloured red or blue. Show that there must exist three equally spaced points of the same colour.