MATH221-001 200630 Group Work Assignment 3: Relations and Equivalence Classes

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Please try to solve as many of the following problems as you can with your group. At the end, hand in one set of answers for everyone in the group. Make sure that everyone in your group puts their name on the answer pages. The problems will be marked, and everyone in the group will get the same grade. You problably won't have time do do all the problems, so select the ones you can do best.

1. For each of the following relations on the set of all living human beings, determine whether the relation is reflexive, symmetric, transitive, and/or an equivalence relation. Clearly state any assumptions you are making.

   (a) $xRy =$ "x is y's mother"

   (b) $xRy =$ "x is y's ancestor"

   (c) $xRy =$ "x and y share the same family (last) name"

   (d) $xRy =$ "x is married to y"

   (e) $xRy =$ "x has the same birthday as y "

   For each of the above relations which are equivalence relations, find the equivalence classes and describe the quotient set.

2. For each of the following relations on the set $\mathbb{N}$, determine whether the relation is reflexive, symmetric, transitive, and/or an equivalence relation.

   (a) $xRy =$ "x is a multiple of y"

   (b) $xRy =$ "$x+y$ is a multiple of 3"

   (c) $xRy =$ "$x+2y$ is a multiple of 3"

   (d) $xRy =$ "xy is a multiple of 3"

   (e) $xRy =$ "$x = ny$ where n is some integer power of 10, i.e., $n \in \{1, 10, 100, 1000, \ldots\}$"

   (f) $xRy =$ "$x = ny$ or $y = nx$ where n is some power of 10.

   (g) $xRy =$ "x has the same last 2 digits as y"

   For each of the above relations which are equivalence relations, find the equivalence classes and describe the quotient set.

3. For any relation R, we can define the relation $\not R$ by $x \not R y$ if and only if $\neg(xRy)$. Consider, for example the relation $=$ and th corresponding relation $\neq$. For each of the relations in questions 1 and 2, decide whether $\not R$ is symmetric, reflexive, and/or transitive. Can you see any patterns?

4. **Efron's Dice.** I have a weird set of three dice, one coloured red, one coloured green, and one coloured blue. The red one, instead of having the numbers $1, \ldots, 6$ on its faces, has the numbers $1, 4, 4, 4, 4, 4$. Furthermore, the blue one has the numbers $3, 3, 3, 3, 3, 6$, and the green one has the numbers $2, 2, 2, 5, 5, 5$. I want to know which of these dice is the best, so I define a relation "B" on the set of these three dice by $d_1 B d_2 =$ "d_1 ties or beats d_2 on average". Determine whether the relation is reflexive, symmetric, and/or transitive.

5. Is it true that a relation which is symmetric and transitive must be reflexive?

6. A relation is called "total" if for all x there is a y such that xRy.

   (a) Give an example of a total relation.

   (b) Give an example of a relation which is not total.

   (c) Give an example of a relation which is both transitive and total.

   (d) Give an example of a relation which is transitive, total, and irreflexive (i.e., for all x it is not the case that xRx).

   (e) Show that there is no example of a relation which is transitive, total, and irreflexive on a finite set.