UNIVERSITY OF REGINA
DEPARTMENT OF MATHEMATICS AND STATISTICS
MATH 221–001 200630

Midterm Test 1

Time: 50 minutes	Name: ____________________
Instructor:	Student #: ____________________
Dr. Edward Doolittle	Section: ____________________

You have 50 minutes to do each of the following questions. The test is worth a total of 50 marks; you should try to earn one mark per minute. Please justify all your answers with proofs or examples, as appropriate. No aids are permitted. Use the backs of the pages for rough work.

The last two problems are much harder than the others. You should attempt them only after you have tried the others. If hardly anyone gets the last two problems, the test will be take out of 40 or 45 instead of out of 50.

1. (a) (5 marks) Show that the logical expressions $\neg(\neg p \vee q)$ and $\neg q \wedge p$ are equivalent.

   (b) (5 marks) Show that the logical expression $\neg q \wedge p \Leftrightarrow \neg(\neg p \vee q)$ is a tautology.

(c) (5 marks) Show that, for any two sets A and B, $B^c \cap A = (A^c \cup B)^c$.

2. (5 marks) Negate the following sentence: If it is raining or snowing, then it is cloudy and cold. Do not just prepend “it is not the case that” to the beginning of the sentence.

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3. (5 marks) Consider the function $f : \mathbb{N} \longrightarrow \mathbb{N}$ defined by

$$f(n) = \begin{cases} n+1, & n \text{ odd} \\ n-1, & n \text{ even} \end{cases}$$

Determine whether f is an injection, surjection, and/or bijection.

4. Suppose $f : S \longrightarrow S$ is a surjection, but $g : S \longrightarrow S$ is not.

   (a) (5 marks) Prove that $g \circ f$ is never a surjection.

(b) (5 marks) Give an example of functions $f : S \to S$ and $g : S \to S$ where f is a surjection, g is not a surjection, yet $f \circ g$ is a surjection. (Hint: try an infinite set S.)

5. (5 marks) Find sets A, B, and C which are subsets of $U = \mathbb{N}$ and for which the set relation

$$A \cap (B \cup C) = (A \cap B) \cup C$$

is false.

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6. (5 marks) Is the function $g : \mathbb{N} \to \mathbb{N}$ defined by

$$g(n) = \begin{cases} n^2 + 3 & n \text{ odd} \\ 4n + 8 & n \text{ even} \end{cases}$$

an injection?

7. (5 marks) Show that, if $f : S \to S$ is a bijection on a *finite* set S, then there must be a natural number k such that $f^k(x) = x$ for all $x \in S$. Bonus: what can you say if S is infinite?