UNIVERSITY OF REGINA
DEPARTMENT OF MATHEMATICS AND STATISTICS
MATH 221–001 200630

Midterm Test 2

Time: 50 minutes
Instructor:
Dr. Edward Doolittle

Name: _____
Student #: _____
Section: _____

You have 50 minutes to do each of the following questions. The test is worth a total of 50 marks; you should try to earn one mark per minute. Please justify your conclusions with a proof or an example. No aids are permitted. Use the backs of the pages for rough work.

The last two problems are harder than the others. You should attempt them only after you have tried the others. If hardly anyone gets the last two problems, the test will be taken out of 40 or 45 instead of out of 50.

1. (a) (5 marks) Show that if I select 56 distinct numbers from the set $\{1, 2, \ldots, 100\}$ I must have two numbers which differ by 11.

(b) (5 marks) Show that if I select 55 distinct numbers from the set $\{1, 2, \ldots, 100\}$ then I do not necessarily have two numbers which differ by 11.

2. (6 marks) Recall that we proved the formulas

$$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$$
$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$

Use those formulas to find a similar formula for

$$\sum_{r=1}^{n} (3r^2 + 7r)$$

You do not need to prove that your formula is correct!

3. (6 marks) Use mathematical induction to prove that

$$\sum_{r=1}^{n}(3r^2+5r) = n(n+1)(n+3)$$

for $n = 1, 2, 3, \ldots$.

4. (6 marks) Recall that the Fibonacci numbers are defined by $f_1 = f_2 = 1$ and $f_{n+2} = f_{n+1} + f_n$ for $n = 1, 2, \ldots$. Prove that

$$f_1^2 + f_2^2 + \cdots + f_n^2 = f_n f_{n+1}$$

for all $n \in \mathbb{N}$.

5. (6 marks) Show that $2^n > 2n+1$ for $n = 3,4,5,\ldots$. (Recall that 2^n is defined inductively by $2^1 = 2$ and $2^{n+1} = 2 \times 2^n$.)

6. (6 marks) Show that the set $\{n(n+1)(n+3) : n \in \mathbb{N}\} = \{8,30,72,\ldots\}$ is countable. (Hint: you may find question 3 useful.)

7. (5 marks) I pick three distinct numbers from the set $\{1, 2, \ldots, 10\}$ and add them up. For example, I could pick 3, 7, and 8 and get the result 18. Suppose I repeat that procedure 23 times. Show that I must have some result twice.

8. (5 marks) Show that the set of primes of the form $3n + 2$ is infinite.