MATH221-001 200630 Problem Set 1

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1. Show that the expressions $(p \Rightarrow q) \wedge (q \Rightarrow p)$ and $p \Leftrightarrow q$ are logically equivalent.
2. Draw up truth tables for each of the following expressions:

   (a) $\neg(p \vee q) \wedge r$ (b) $\neg((p \vee q) \wedge \neg r)$

   Are the two expressions logically equivalent? If not, find an assignment of truth values to p, q, and r which proves that the expressions are not equivalent, and construct English sentences illustrating the expressions in that case.
3. (Reductio ad absurdum.) Show that the following statement is a tautology: $((\neg p \Rightarrow q) \wedge (\neg p \Rightarrow \neg q)) \Rightarrow p$.
4. Find a converse, an inverse, and a contrapositive for the statement "If Curious George is a monkey and 2 is a prime, then 2 is a monkey."
5. Negate the following sentence: "Private schools implementing a voucher system will fail to provide equal educational opportunities across a community if they either skim off the best students and leave the poorer ones behind or they charge parents fees beyond what is paid for in private funds and so exclude children of poorer families." You should not simply tack the phrase "it is not the case that" to the beginning.
6. The island of Smulland is undergoing a census. A man responds to the census taker, "If I am a knight, then so is my wife." Is it possible to tell whether the husband is a knight or a knave? Is it possible to tell whether the wife is a knight or a knave?
7. Translate each of the following into logical notation. (Time is an implicit factor in these statements, so each should begin with $\forall t$.)

   (a) No dogs must be brought into this park except on a leash.
   (b) Dogs are not allowed into this park without leash.
   (c) Owners of dogs are not allowed into this park unless they keep them on leashes.
   (d) Nobody without his dog on a leash is allowed in this park.
   (e) Dogs must be walked on a leash in this park.
   (f) All dogs must be kept on leashes in this park.

   Can you come up with a better alternative?
8. Consider three barbers in a shop, Allen, Brown, and Carr, whose movements always satisfy the following conditions: (1) if Allen goes out then Brown has to go out too (because Allen is infirm); (2) all three cannot be out together (for business reasons).

   (a) Is it possible for Carr to go out?
   (b) Is condition (2) is equivalent to the following proposition: "If Carr goes out, then if Allen goes out, then Brown stays in."
   (c) Show that the proposition "If Allen goes out, then Brown stays in" is false.
   (d) Use the contrapositive of the statement in (b) to prove that Carr never goes out.

   Can you resolve the paradox?