MATH221-001 200630 Problem Set 4 Solutions DRAFT

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1. The pigeonholes are $\{1, 99\}$, $\{2, 98\}$, $\{3, 97\}$, ..., $\{49, 51\}$, and 50 and 100 are left over and don't help us so they should go in pigeonholes by themselves: $\{50\}$ and $\{100\}$. Altogether we have $49+1+1 = 51$ pigeonholes, so any choice of 52 numbers must include two from the same pigeonhole, i.e., two numbers that add up to 100.

2. (a) The pigeonholes are $\{1, 2\}$, $\{3, 4\}$, ..., $\{19, 20\}$, 10 in all, so any choice of 11 numbers must include two from the same pigeonhole, i.e., two that differ by 1.

   (b) The pigeonholes are $\{1, 3\}$, $\{2, 4\}$, $\{5, 7\}$, $\{6, 8\}$, $\{9, 11\}$, $\{10, 12\}$, $\{13, 15\}$, $\{14, 16\}$, $\{17, 19\}$, $\{18, 20\}$, 10 in all, so any choice of 11 numbers must include two from the same pigeonhole.

3. There are 1000 different three digit numbers from 000 to 999, so if I have 1001 license plates, the three digit numbers are guaranteed to be the same in two of them. Actually, there may be restrictions on the possible three digit numbers which reduce the number of possibilities below 1000 (e.g., 000 may not be possible; there certainly are restrictions on the letter part of the plate); I don't know enough about it to say. But 1000 is guaranteed to be the maximum number of possibilities.

4. The number should have been 5 people with the same last initial, but the given statement is still true. (Why?) There are 26 possible last initials (assuming the English alphabet), giving us 26 different pigeonholes. If we had no more than 4 people with the same last initial, we could have no more than $4 \times 26 = 104$ people altogether; but we have 105, so our assumption that no more than 4 people have the same last initial must be false.

5. This is similar to the previous, with $100,000$ pigeonholes (someone could have 0 hairs on their head) and over $10,000 \times 100,000 = 10^9$ people.

6. The values make 13 pigeonholes, so I need 14 cards to guarantee a pair.

7. The answer is $13 \times 3 + 1 = 40$ cards, by reasoning similar to question 4.

8. If we had a fruit basket which didn't meet Mick's requirements, we could have no more than 6 apples, 4 bananas, and 9 oranges, or no more than 19 pieces of fruit. So if I tell my assistant to get 20 pieces of fruit, I'm guaranteed to have at least 7 apples, or 5 bananas, or 10 oranges.

9. Let's say there were n players. The pigeonholes are the number of games won. Each player played $n - 1$ other players, so could have won at most $n - 1$ games. On the other hand, we're told that every player won at least 1 game, so there are $n - 1$ pigeonholes (for players that won 1 game, 2 games, 3 games, ..., $n - 1$ games respectively). Since there are n players fitting into $n - 1$ pigeonholes, we know that 2 players must go into the same hole, i.e., have the same number of games.

10. Here the pigeonholes are complicated. They are $\{1, 2, 4, 8, 16, 32, 64\}$, $\{3, 6, 12, 24, 48, 96\}$, $\{5, 10, 20, 40, 80\}$, $\{7, 14, 28, 56\}$, etc., i.e., we start with an odd number and keep multiplying by 2 until we go above 100. For any two numbers in one of those pigeonholes, one number divides the other (with a quotient of some power of 2); try picking two numbers in any one of the pigeonholes and see what I mean. There are 50 pigeonholes indexed by the odd numbers from 1 to 99, so if I have 51 numbers, two must come from the same pigeonhole, and I'm done.