MATH221-001 200630 Problem Set 4

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Due: Monday, November 6

1. Suppose I select a set of 52 numbers from the set $\{1,\dots,100\}$. Show that I must have a pair of numbers the sum of which is 100.

2. Show that if I select 11 natural numbers from the set $\{1,\dots,20\}$, I must have

   (a) Two numbers which differ by 1; and

   (b) Two numbers which differ by 2.

3. How many car owners do I need in to guarantee that two of them have the same three digit number in their Saskatchewan license plate?

4. Show that in any group of 105 people, there must be four with the same last initial.

5. Show that there must be a group of $10,000$ people in China all with the same number of hairs on their heads. (There are over a billion people in China; the human head can have up to $99,999$ hairs.)

6. In a standard deck of 52 cards (13 values, 4 suits), what is the smallest number of cards you need to draw to guarantee that you have a pair (same value, different suits)?

7. In a standard deck of 52 cards (13 values, 4 suits), what is the smallest number of cards you need to draw to guarantee you have 4 cards of one suit?

8. **(Strong pigeonhole principle.)** The musician Mick Richards is very particular about his pre-concert fruit basket. He requires that it contain at least 7 apples, *or* at least 5 bananas, *or* at least 10 oranges. I'd like to keep Mick happy but my assistant failed logic, so I want a simpler way of guaranteeing that Mick's requirements are met. I want to just say to my assistant, "Go to the fruit store (which sells only apples, bananas, and oranges) and make a basket of *n* pieces of fruit." What value of *n* will guarantee that Mick's requirements are met?

9. In a round-robin tournament (where each player plays each other player), everyone won at least one game. Show that there must be two players with the same number of wins.

10. Suppose I select a set of 51 numbers from the set $\{1,\dots,100\}$. Show that I must have two numbers such that one divides the other.