MATH221-001 200630 Problem Set 5

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Due: Wednesday, November 29

1. Show that the relation R used in the definition of $\mathbb{Z}$, i.e., $[a,b]R[c,d]$ if and only if $a+d=b+c$, is an equivalence relation.

2. Show that the operations $-$ and $\times$ defined by $[a,b]-[c,d]=[a+d,b+c]$ and $[a,b]\times[c,d]=[ac+bd,ad+bc]$ are independent of representatives, i.e., are well-defined on $\mathbb{Z}$.

3. (a) Find $d=\text{GCD}(2990,2622)$.
   (b) Find integers m and n such that $d=2990m+2622n$.
   (c) Find a different pair of integers m' and n' such that $d=2990m'+2622n'$.
   (d) Find integers m_1 and n_1 such that $2990m_1+2622n_1=230$.
   (e) Is it possible to find integers m_2 and n_2 such that $2990m_2+2622n_2=235$?

4. At a clambake, the cost of a lobster dinner is \$11 and the cost of a chicken dinner is \$8. What can you conclude if the total bill is

   (a) \$777 (b) \$96 (c) \$69

5. Find all prime numbers between 1 and 200.

6. Show that the n consecutive positive integers beginning with $(n+1)!+2$ are all composite.

7. Let a,b,d be integers with $\text{GCD}(a,b)=d$. Show that $\text{GCD}(a/d,b/d)=1$.

8. Show that if a,b are relatively prime integers, then $\text{GCD}(a+b,a-b)=1$ or 2.

9. How many ways can change for a dollar be made using

   (a) dimes and quarters
   (b) nickels, dimes, and quarters
   (c) pennies, nickels, dimes, and quarters?

10. Is it possible to have 50 coins, all pennies, dimes, and quarters, worth \$3?