MATH221-001 200630 Problem Set 6

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You do not need to hand this problem set in!

The purpose of this problem set is to help you learn the basic ideas of modular arithmetic. You do not need to hand it in!

1. Find the following residues:

   (a) $100 \mod 13$ (b) $123456789 \mod 9$ (c) $9652 \mod 1000$ (d) $123456789 \mod 11$

2. Without multiplying more than two 2-digit numbers in each case, show that

   (a) $17263 \times 19274 \equiv 2 \mod 10$
   (b) $392 \times 36127 \equiv -1 \mod 5$
   (c) $2366 \times 5003 \equiv -2 \mod 25$
   (d) $839 \times 5923 \equiv 13 \mod 8$

3. Using arithmetic modulo 10, find the last digit of the following numbers:

   (a) 3^4 (b) 7^{40} (c) 217^{40} (d) 2^{157}

4. Use the method of casting out 9s to determine that two of the following statements is false. (Casting out 9s does not guarantee that a result is true, but identifies certain false results with certainty; it is most effective at identifying single digit errors; it doesn't help at all in identifying transposition errors.)

   (a) $8,901 \times 5,743 = 51,181,443$
   (b) $9,787 \times 1,258 = 12,310,046$
   (c) $6,893 \times 16,922 = 115,543,346$
   (d) $5,783 \times 40,162 = 232,256,846$

5. Use divisibility tests to show that none of the following numbers is prime.

   (a) $52,739,253$ (b) $391,391$ (c) $39,360,711$ (d) $19,392,329$

6. Let x, y, z, w be integers such that $x^2 + y^2 - 3z^2 - 3w^2 = 0$. Use arithmetic mod 3 to show that all of x, y, z, w must be 0.

7. Construct a round robin tournament for 7 players.

8. Find the inverses of

   (a) $2 \mod 11$ (b) $7 \mod 15$ (c) $7 \mod 16$ (d) $5 \mod 13$

9. Suppose $GCD(r, m) = 1$. Formulate a method based on the Euclidean Algorithm for finding the inverse of r modulo m. Use your method to find the inverse of $47 \mod 256$.

10. Prove that the congruence $ax \equiv b \mod m$ has a solution x if and only if b is a multiple of $GCD(a, m)$.