

UNIVERSITY OF REGINA
DEPARTMENT OF MATHEMATICS AND STATISTICS
MATH 281 200610 Midterm Test 1
Edward Doolittle

Time: 50 minutes

Name: _____

Instructor: Dr. Edward Doolittle

Student #: _____

(marks) Please do all questions. You have 10 minutes to do each question, for a total of 50 minutes in 50 minutes for the quiz. A non-programmable calculator is allowed but is not necessary. If you finish early, I recommend you check your work thoroughly.

(10) 1. Show that the implicitly-defined function

$$-2x^2y + y^2 = 1$$

is a solution to the differential equation

$$2xy \, dx + (x^2 - y) \, dy = 0.$$

Find at least one explicit solution. Give an interval of definition of your explicit solution

(10) 2. Describe what is meant by an autonomous differential equation. Sketch a phase portrait for the autonomous equation

$$\frac{dy}{dx} = y^2 - 3y.$$

Identify the equilibrium points and indicate whether each of the equilibrium points is stable or unstable.

(10) 3. Solve the differential equation

$$\frac{dy}{dx} = x\sqrt{1 - y^2}.$$

Find at least one explicit solution (and interval of definition).

(10) 4. Solve the initial value problem

$$(x + 1)\frac{dy}{dx} + y = \ln x, \quad y(1) = 10.$$

(10) 5. Solve the exact differential equation

$$(x - y^3 + y^2 \sin x) \, dx = (3xy^2 + 2y \cos x) \, dy.$$

Leave your solution in implicit form.