

MATH281 200610 Problem Set 1

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The following problems from the exercises in section 1.1 may appear on the quiz on January 25.

1. (Based on 1.1.1-8) State the order of the given ordinary differential equation. Determine whether the equation is linear or nonlinear.

(a) $x \frac{d^3 y}{dx^3} - \left(\frac{dy}{dx}\right)^4 + y = 0$

(b) $\frac{d^2 R}{dt^2} = -\frac{k}{R^2}$

(c) $(\sin \theta)y''' - (\cos \theta)y' = 0$ where y is a function of θ .

2. (Based on 1.1.11-18) Verify that the indicated function is a solution of the given ordinary differential equation. Find an interval I of definition of the solution.

(a) $\frac{dy}{dt} + 20y = 24$; $y = \frac{6}{5} - \frac{6}{5}e^{-20t}$

(b) $y' = 25 + y^2$; $y = 5 \tan 5x$

(c) $2y' = y^3 \cos x$; $y = (1 - \sin x)^{-1/2}$.

3. (Based on 1.1.19-24) Verify that the indicated function is a solution of the given ordinary differential equation. If necessary, use implicit differentiation to verify that the indicated expressions define a solution to the given differential equations. Find an interval I of definition of the solution. If applicable, find an explicit solution (a function in explicit form) for the differential equation. (The parameters c_i are constants.)

(a) $2xy \, dx + (x^2 - y) \, dy = 0$; $-2x^2y + y^2 = 1$

(b) $\frac{dy}{dx} + 2xy = 1$; $y = e^{-x^2} \int_0^x e^{t^2} \, dt + c_1 e^{-x^2}$

(c) $x^3 \frac{d^3 y}{dx^3} + 2x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = 12x^2$; $y = c_1 x^{-1} + c_2 x + c_3 x \ln x + 4x^2$

4. (Based on 1.1.25) Verify that the piecewise-defined function

$$y = \begin{cases} -x^2, & x < 0 \\ x^2, & x \geq 0 \end{cases}$$

is a solution of the differential equation $xy' - 2y = 0$ on the interval $(-\infty, \infty)$. Can you find any other solutions to the differential equation?

5. (Based on 1.1.28) Find values of m so that the function $y = x^m$ is a solution of the given differential equation. Explain your reasoning.

(a) $xy'' + 2y' = 0$

(b) $x^2 y'' - 7xy' + 15y = 0$

For additional practice, you should try every problem in the list 1.1.1-32.