MATH281 200610 Problem Set 2

Edward Doolittle

Wednesday, January 18, 2006

The following problems from the exercises in sections 1.2 and 1.3 may appear on the quiz on January 25.

1. (Based on 1.2.2, 1.2.4, and 1.2.10) Find solutions to the following initial value problems.
   (a) $y' = y - y^2$, $y(-1) = 2$. (Hint: $y = 1/(1 + ce^{-x})$ is a general solution to the DE.)
   (b) $y' + 2xy^2 = 0$, $y(-2) = 1/2$. (Hint: $y = 1/(x^2 + c)$ is a general solution to the DE.)
   (c) $x'' + x = 0$, $x(\pi/4) = \sqrt{2}$, $x'(\pi/4) = 2\sqrt{2}$. (Hint: $x = c_1 \cos t + c_2 \sin t$ is a general solution to the DE.)
2. (Based on 1.2.20, 1.2.22, and 1.2.24) Determine a region of the xy-plane for which the given differential equation would have a unique solution the graph of which passes through a point (x_0, y_0) in the region.
   (a) $\dfrac{dy}{dx} - y = x$
   (b) $(1 + y^3)y' = x^2$
   (c) $(y - x)y' = y + x$.
3. (Based on 1.2.26 and 1.2.28) Determine whether Theorem 1.1 guarantees that the differential equation $y' = \sqrt{y^2 - 9}$ possesses a unique solution through the given point.
   (a) $(5, 3)$
   (b) $(-1, 4)$
4. (Based on 1.2.30)
   (a) Verify that $y = \tan(x + c)$ is a one-parameter family of solutions of the differential equation $y' = 1 + y^2$.
   (b) Find the largest region in the xy-plane for which Theorem 1.1 guarantees that the differential equation would have a unique solution the graph of which passes through a point (x_0, y_0) in the region.
   (c) Use the given one-parameter family of solutions to find an explicit solution to the IVP $y' = 1 + y^2$, $y(0) = 0$.
   (d) Even though x_0 in the above solution is in the interval $-2 < x < 2$, the above solution is not defined on that interval. Explain why that does not contradict Theorem 1.1.
   (e) Determine the largest interval I of definition for the solution to the IVP given above.
5. (Based on 1.3.24) Suppose a hole is drilled through the center of the earth and a ball is dropped into the hole. Construct a mathematical model that describes the motion of the ball. (Hint: think of the earth as a series of concentric spherical shells; Newton tells us that the gravitational force due to a spherical shell on a point outside the shell is the same as if all the mass of the shell were concentrated in the center, whereas the force due to a spherical shell on a point inside the shell is zero.)

For additional practice, you should try every problem in the list 1.2.1–34. Chapter 1.3 will be de-emphasized so you need not spend too much time studying it if you are not interested in the material.