

MATH281 200610 Problem Set 3

Edward Doolittle

Wednesday, January 25, 2006

1. (Based on 2.1.22, 2.1.24, and 2.1.26) Find the critical points and phase portrait of the given autonomous first-order differential equation. Classify each critical point as stable or unstable. Sketch typical solution curves in the regions of the xy -plane determined by the graphs of the equilibrium solutions.

(a) $\frac{dy}{dx} = y^2 - y^3$

(b) $\frac{dy}{dx} = 10 + 3y - y^2$

(c) $\frac{dy}{dx} = y(2 - y)(4 - y)$

2. (Based on 2.1.29 and 2.1.30) Consider the autonomous differential equation $dy/dx = f(y)$ where the graph of f is given in two different cases in Figure 1. Use the graph to locate the critical points of each differential equation. Sketch a phase portrait of each differential equation. Which critical points are stable?

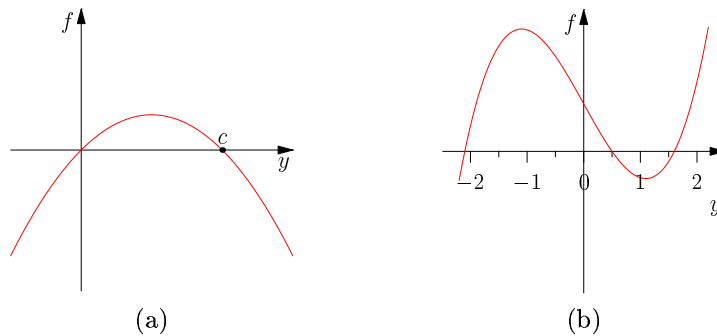


Figure 1: Graphs for Problem 2

3. (Based on 2.2.4, 2.2.8, and 2.2.14) Solve the following differential equations by separation of variables.

(a) $dy - (y - 1)^2 dx = 0$

(b) $e^x yy' = e^{-y} + e^{-2x-y}$

(c) $\frac{dy}{dx} = \frac{x(1 + y^2)^{1/2}}{y(1 + x^2)^{1/2}}$

4. (Based on 2.2.24, 2.2.26, and 2.2.28) Solve the following initial value problems.

(a) $\frac{dy}{dx} = \frac{y^2 - 1}{x^2 - 1}$, $y(2) = 2$

(b) $\frac{dy}{dt} + 2y = 1$, $y(0) = 5/2$

(c) $\frac{dy}{dx} = \frac{x(1 + 4y^2)}{-(1 + x^4)}$, $y(1) = 0$

5. (Based on 2.2.32) Check that an implicit solution to the equation $2x \sin^2 y dx - (x^2 + 10) \cos y dy = 0$ is given by $\ln(x^2 + 10) + \csc y = c$. Identify any lost solutions which cannot be obtained from the implicit solution.

For additional practice, you should try problems 2.1.19–30 and 2.2.1–32. The questions in chapter 2.2 will tax your capabilities in integration; in particular, you may have to look up in your first year calculus textbook how to integrate $\operatorname{sech} x$, the hyperbolic secant function. If you are handy with computers, you should try the other problems in chapters 2.1 and 2.2, but you won't be tested on the material.