MATH281 200610 Problem Set 4

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1. (Based on 2.3.10, 2.3.16, and 2.3.18.) Solve the following linear differential equations.

   (a) $x\dfrac{dy}{dx} + 2y = 3$ (b) $y\,dx = (ye^y - 2x)\,dy$ (c) $(\tan x)y' + y = \sec^3 x$

2. (Based on 2.3.26, 2.3.28, and 2.3.30.) Solve the following initial value problems.

   (a) $y\dfrac{dx}{dy} - x = 2y^2,\ y(1) = 2$ (b) $\dfrac{dT}{dt} = k(T - T_m),\ T(0) = T_0$ (c) $\dfrac{y' + (\tan x)y}{\cos^2 x} = 1,\ y(0) = 1$

3. (Based on 2.3.32 and 2.3.34.) Solve the following initial value problems with discontinuous coefficients.

   (a) $\dfrac{dy}{dx} + y = f(x),\ y(0) = 1$, where

   $$f(x) = \begin{cases} 1, & 0 \le x \le 1 \\ -1, & 1 < x \end{cases}$$

   (b) $(1 + x^2)\dfrac{dy}{dx} + 2xy = f(x),\ y(0) = 0$, where

   $$f(x) = \begin{cases} x, & 0 \le x < 1 \\ -x, & 1 \le x \end{cases}$$

4. (Based on 2.4.2, 2.4.14, 2.4.32, and 2.4.34.) Solve the following differential equations. You may need to find an appropriate integrating factor.

   (a) $(2x + y)dx + (x + 6y)dy = 0$

   (b) $\left(1 - \dfrac{3}{y} + x\right)\dfrac{dy}{dx} + y = \dfrac{3}{x} - 1$

   (c) $y(x + y + 1)dx + (x + 2y)dy = 0$

   (d) $\cos x\,dx + \left(1 + \dfrac{2}{y}\right)\sin x\,dy = 0$

5. (Based on 2.4.22, 2.4.24, 2.4.38, and 2.4.26.) Solve the following initial value problems. You may need to find an appropriate integrating factor.

   (a) $(e^x + y)\,dx + (2 + x + ye^y)\,dy = 0,\ y(0) = 1$

   (b) $(6y^2 - 2t^2)\dfrac{dy}{dt} + ty = 0,\ y(1) = 1$

   (c) $(x^2 + y^2 - 5)dx = (y + xy)dy,\ y(0) = 1$

   (d) $\left(\dfrac{1}{1 + y^2} + \cos x - 2xy\right)\dfrac{dy}{dx} = y(y + \sin x),\ y(0) = 1$

For additional practice you should try problems 2.3.1–37 (no need to do the graphing) and 2.4.1–39 (again, no need to do the graphing). Problems for midterm test 1 will be selected from the additional practice suggestions in problem sets 1–4.