MATH281 200610 Problem Set 5 DRAFT

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1. (Based on 4.1.2, 4.1.4, and 4.1.6.) Check that the given family of functions is a solution to the following differential equations on the given intervals, and then solve the indicated initial value problems.

   (a) $y = c_1 e^{4x} + c_2 e^{-x}$, $I = (-\infty, \infty)$; $y'' - 3y' - 4y = 0$; $y(0) = 1$, $y'(0) = 2$

   (b) $y = c_1 + c_2 \cos x + c_3 \sin x$, $I = (-\infty, \infty)$; $y''' + y' = 0$; $y(\pi) = 0$, $y'(\pi) = 2$, $y''(\pi) = -1$

   (c) $y = c_1 + c_2 x^2$, $I = (-\infty, \infty)$; $xy'' - y' = 0$; $y(0) = 0$, $y'(0) = 0$

   Discuss your results in light of Theorem 4.1.

2. (Based on 4.1.12 and 4.1.14.) Use the families of functions given in question 1 to solve the following boundary value problems.

   (a) $y'' - 3y' - 4y = 0$; $y(0) = 0$, $y(1) = \sinh(5/2)$

   (b) $xy'' - y' = 0$; $y(0) = 1$, $y'(1) = 6$ (the boundary condition is "mixed" because it involves the value of the function and the derivative of the function)

3. (Based on 4.1.16, 4.1.18, and 4.1.22.) Determine whether the given set of functions is linearly independent on the interval $I = (-\infty, \infty)$.

   (a) $f_1(x) = 0$, $f_2(x) = x$, $f_3(x) = e^x$

   (b) $f_1(x) = \cos 2x$, $f_2(x) = 1$, $f_3(x) = \cos^2 x$

   (c) $f_1(x) = e^x$, $f_2(x) = e^{-x}$, $f_3(x) = \sinh x$

4. (Based on 4.1.26, 4.1.28, and 4.1.30.) Verify that the given functions form a fundamental set of solutions of the given equation on the indicated interval. Form the general solution.

   (a) $4y'' - 4y' + y = 0$; $e^{x/2}$, $xe^{x/2}$, $I = (-\infty, \infty)$

   (b) $x^2 y'' + xy' + y = 0$; $\cos(\ln x)$, $\sin(\ln x)$, $I = (0, \infty)$

   (c) $y''' + y' = 0$; 1, $\cos x$, $\sin x$, $I = (-\infty, \infty)$

5. Guess particular solutions to the following differential equations, and then use the results of question 4 to form the general solution.

   (a) $4y'' - 4y' + y = 7$

   (b) $x^2 y'' + xy' + y = x^2$

   (c) $y''' + y' = 1 + e^x$

   Hint: in each case, the function y is very similar to the function f on the right side of the equation.

For additional practice you should try problems 4.1.1–36. To solve some of the problems (e.g., 4.1.20) you will have to think "outside the box".