

MATH281 200610 Problem Set 7 DRAFT

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1. (Based on 4.4.4, 4.4.12, and 4.4.26.) Solve the following problems by the method of undetermined coefficients (superposition approach).

(a) $y'' + y' - 6y = 2x$

(b) $y'' - 16y = 2e^{4x}$

(c) $y^{(4)} - y'' = 4x + 2xe^{-x}$

2. (Based on 4.4.28, 4.4.34, 4.4.36.) Solve the following initial-value problems.

(a) $2y'' + 3y' - 2y = -4x - 11$
 $y(0) = 0, y'(0) = -10$

(b) $x''(t) + \omega^2 x(t) = F_0 \cos \gamma t$
 $x(0) = 0, x'(0) = 0$

(c) $y''' + 8y = 2x - 5 + 8e^{-2x}$
 $y(0) = 0, y'(0) = -5, y''(0) = -4$

3. (Based on 4.4.38 and 4.4.40.) Solve the following boundary-value problems.

(a) $y'' - 2y' + 2y = 2x - 2$
 $y(0) = 0, y'(\pi) = \pi$

(b) $y'' + 3y = 6x$
 $y(0) + y'(0) = 0, y(1) = 0$

4. (Based on 4.4.42.) Solve the initial value problem $y'' - 2y' + 10y = g(x)$, $y(0) = 0$, $y'(0) = 0$ with discontinuous "input" function

$$g(x) = \begin{cases} 20, & 0 \leq x \leq \pi \\ 0, & x > \pi \end{cases}$$

5. (Based on 4.5.16, 4.5.22, 4.5.24, and 4.5.26.) Find a constant coefficient linear ordinary differential operator that annihilates the given function.

(a) $x^3(1 - 5x)$

(b) $8x - \sin x + 2 \cos 5x$

(c) $(2 - e^x)^2$

(d) $e^{-x} \sin x - e^{2x} \cos x$

6. (Based on 4.5.28, 4.5.30, 4.5.32, and 4.5.34.) Identify the order n of each of the following linear differential operators, and for each of the operators find n linearly independent functions that are annihilated by the operator.

(a) $D^2 + 4D$

(b) $D^2 - 9D - 36$

(c) $D^2 - 6D + 10$

(d) $D^2(D - 5)(D - 7)$

7. (Based on 4.5.40, 4.5.50, and 4.5.52.) Solve the following differential equations by undetermined coefficients (annihilator approach).

(a) $y'' + 3y' = 4x - 5$

(b) $y'' + 3y' - 10y = x(e^x + 1)$

(c) $y'' + 2y' + y = x^2e^{-x}$

For additional practice you should try problems 4.4.1–42 and 4.5.1–72.