MATH281 200610 Problem Set 8 DRAFT

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1. (Based on 4.6.2, 4.6.4, and 4.6.6.) Solve the following differential equations for $y(\theta)$ by variation of parameters given that the general solution to $y'' + y = 0$ is $y = c_1 \cos\theta + c_2 \sin\theta$.

   (a) $y'' + y = \tan\theta$ (b) $y'' + y = \sec\theta\tan\theta$ (c) $y'' + y = \sec^2\theta$

2. (Based on 4.6.12 and 4.6.14.) Solve the following differential equations for $y(t)$ by variation of parameters.

   (a) $y'' - 2y' + y = e^t(1+t^2)^{-1}$ (b) $y'' - 2y' + y = e^t \arctan t$ (c) $y'' - 2y' + y = e^t \ln t$

3. (Based on 4.6.10, 4.6.16, and 4.6.18.) Solve the following differential equations by variation of parameters.

   (a) $y'' - 9y = 9xe^{-3x}$ (b) $2y'' + 2y' + y = 4\sqrt{x}$ (c) $4y'' - 4y' + y = e^{x/2}\sqrt{1-x^2}$

4. (Based on 4.6.20 and 4.6.22.) Solve the following differential equations subject to the initial conditions $y(0) = 1$, $y'(0) = 0$:

   (a) $2y'' + y' - y = x + 1$ (b) $y'' - 4y' + 4y = (12x^2 - 6x)e^{2x}$

5. (Based on 4.6.26.) Solve the third-order differential equation $y''' + 4y' = \sec 2x$ by variation of parameters.

6. (Based on 4.6.24.) In this problem we use variation of parameters to find the solution to a variable coefficient LODE.

   (a) Verify that $y_1(x) = \cos(\ln x)$ is a solution to the equation $x^2y'' + xy' + y = 0$.
   (b) Use reduction of order to find a fundamental set of solutions to the equation $x^2y'' + xy' + y = 0$.
   (c) Use variation of parameters to find the general solution to the equation $x^2y'' + xy' + y = \sec(\ln x)$.

7. (Based on 4.6.28.) Use a combination of undetermined coefficients, variation of parameters, and the superposition principle for non-homogeneous equations (Theorem 4.7 on page 136) to solve the equation $y'' - 2y' + y = 4x^2 - 3 + x^{-1}e^x$.

8. (Based on 4.6.30.) In this problem we explore the paradigm for solving linear ordinary differential equations outlined in the lectures. We will solve the equation $x^4y'' + x^3y' - 4x^2y = 1$, beginning with finding a solution to the homogeneous equation by whatever means is available to us.

   (a) Because the powers of x in the coefficients in the equation $x^4y'' + x^3y' - 4x^2y = 0$ are decreasing by 1, it may be possible to find a solution of the form $y_1(x) = x^m$ for some m. Substitute that y_1 into the equation, find a condition on m, and thus find a non-trivial solution to the equation.
   (b) By similar means, or by reduction of order, find the general solution to the homogeneous equation.
   (c) Use variation of parameters to find a solution to the equation $x^4y'' + x^3y' - 4x^2y = 1$.

For additional practice you should try problems 4.6.1–30, 4.R.1–6, 4.R.8–20, and 4.R.25–32. The identifier 4.R refers to the review problems at the end of chapter 4.