MATH281 200610 Problem Set 9 DRAFT

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1. (Based on 6.1.12.) Rewrite the given expression as a single power series in x^k:

$$\sum_{n=2}^{\infty} n(n-1)c_n x^n + 2\sum_{n=2}^{\infty} n(n-1)c_n x^{n-2} + 3\sum_{n=1}^{\infty} nc_n x^n.$$

2. (Based on 6.1.14.) Verify by direct substitution that the power series $y = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n}(n!)^2} x^{2n}$ is a particular solution of the differential equation $xy'' + y' + xy = 0$.

3. (Based on 6.1.18, 6.1.22, 6.1.26.) Find two power series solutions for each of the following differential equations about the ordinary point $x = 0$.

   (a) $y'' + x^2 y = 0$ (b) $y'' + 2xy' + 2y = 0$ (c) $(x^2+1)y'' - 6y = 0$

4. (Based on 6.1.30 and 6.1.32.) Solve the given initial-value problems using power series.

   (a) $(x+1)y'' - (2-x)y' + y = 0$, $y(0) = 2$, $y'(0) = -1$
   (b) $(x^2+1)y'' + 2xy' = 0$, $y(0) = 0$, $y'(0) = 1$

5. (Based on 7.1.2 and 7.1.4.) Use the definition of the Laplace transform to find the Laplace transform of the following functions.

   (a) $f(t) = \begin{cases} 4, & 0 \le t < 2 \\ 0, & 2 \le t \end{cases}$ (b) $f(t) = \begin{cases} 2t+1, & 0 \le t < 1 \\ 0, & 1 \le t \end{cases}$

6. (Based on 7.1.12, 7.1.16, and 7.1.18.) Use the definition of the Laplace transform and (if necessary) integration by parts to find the Laplace transform of the following functions.

   (a) $f(t) = e^{-2t-5}$ (b) $f(t) = e^t \cos t$ (c) $f(t) = t \sin t$

7. (Based on 7.1.24, 7.1.26, and 7.1.30.) Use linearity and a table of Laplace transforms to find the Laplace transform of the following functions.

   (a) $f(t) = -4t^2 + 16t + 9$ (b) $f(t) = (2t-1)^3$ (c) $f(t) = (e^t - e^{-t})^2$

8. (Based on 7.1.38 and 7.1.40.) Use trig identities to find the Laplace transform of the following functions.

   (a) $f(t) = \cos^2 t$ (b) $f(t) = 10\cos\left(t - \frac{\pi}{6}\right)$

For additional practice you should try problems 6.1.1–4, 6.1.9–14, 6.1.17–32, 7.1.1–32, and 7.1.37–42.