

MATH281 200610 Problem Set 10 DRAFT

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1. (Based on 7.2.4, 7.2.10, and 7.2.14.) Use linearity to calculate the following inverse Laplace transforms.

$$(a) \mathcal{L}^{-1} \left\{ \left(\frac{2}{s} - \frac{1}{s^3} \right)^2 \right\} \quad (b) \mathcal{L}^{-1} \left\{ \frac{1}{5s-2} \right\} \quad (c) \mathcal{L}^{-1} \left\{ \frac{1}{4s^2+1} \right\}$$

2. (Based on 7.2.20, 7.2.24, and 7.2.28.) Use partial fractions and linearity to calculate the following inverse Laplace transforms.

$$(a) \mathcal{L}^{-1} \left\{ \frac{1}{s^2+s-20} \right\} \quad (b) \mathcal{L}^{-1} \left\{ \frac{1}{s^4-9} \right\} \quad (c) \mathcal{L}^{-1} \left\{ \frac{s^2+1}{s(s-1)(s+1)(s-2)} \right\}$$

3. (Based on 7.2.34, 7.2.36, and 7.2.40.) Use the Laplace transform to solve the following initial value problems.

$$(a) \begin{aligned} y' - y &= 2 \cos 5t, \\ y(0) &= 0. \end{aligned} \quad (b) \begin{aligned} y'' - 4y' &= 6e^{3t} - 3e^{-t}, \\ y(0) &= 1, y'(0) = -1 \end{aligned} \quad (c) \begin{aligned} y''' + 2y'' - y' - 2y &= \sin 3t, \\ y(0) = y'(0) &= 0, y''(0) = 1 \end{aligned}$$

4. (Based on 7.3.6, 7.3.8, 7.3.10.) Use the formula for translation on the s -axis to find the following Laplace transforms.

$$(a) \mathcal{L} \{ e^{2t}(t-1)^2 \} \quad (b) \mathcal{L} \{ e^{-2t} \cos 4t \} \quad (c) \mathcal{L} \left\{ e^{3t} \left(9 - 4t + 10 \sin \frac{t}{2} \right) \right\}$$

5. (Based on 7.3.14, 7.3.16, and 7.3.20.) Use the formula for translation on the s -axis to find the following inverse Laplace transforms.

$$(a) \mathcal{L}^{-1} \left\{ \frac{1}{s^2+2s+5} \right\} \quad (b) \mathcal{L}^{-1} \left\{ \frac{2s+5}{s^2+6s+34} \right\} \quad (c) \mathcal{L}^{-1} \left\{ \frac{(s+1)^2}{(s+2)^4} \right\}$$

6. (Based on 7.3.38, 7.3.40, and 7.3.42.) Use the formula for translation on the t -axis to calculate the following Laplace transforms. Here H is the Heaviside step function.

$$(a) \mathcal{L} \{ e^{2-t} H(t-2) \} \quad (b) \mathcal{L} \{ (3t+1) H(t-1) \} \quad (c) \mathcal{L} \{ \sin t H(t-\pi/2) \}$$

7. (Based on 7.3.44, 7.3.46, and 7.3.48.) Use the formula for translation on the t -axis to calculate the following inverse Laplace transforms.

$$(a) \mathcal{L}^{-1} \left\{ \frac{(1+e^{-2s})^2}{s+2} \right\} \quad (b) \mathcal{L}^{-1} \left\{ \frac{se^{-\pi s/2}}{s^2+4} \right\} \quad (c) \mathcal{L}^{-1} \left\{ \frac{e^{-2s}}{s^2(s-1)} \right\}$$

For additional practice you should try problems 7.2.1–40, 7.3.1–30, and 7.3.37–48.