

# MATH281 200610 Problem Set 11 DRAFT

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1. (Based on 7.3.56, 7.3.60, and 7.3.62.) Graph each of the following functions and then find its Laplace transform.

$$(a) f(t) = \begin{cases} 1, & t \in [0, 4) \\ 0, & t \in [4, 5) \\ 1, & t \in [5, \infty) \end{cases} \quad (b) f(t) = \begin{cases} \sin t, & t \in [0, 2\pi) \\ 0, & t \in [2\pi, \infty) \end{cases} \quad (c) f(t) = \lfloor t \rfloor$$

Here  $\lfloor t \rfloor$  is the floor of  $t$ , i.e., the greatest integer less than or equal to  $t$ .

2. (Based on 7.3.64, 7.3.66, and 7.3.68.) Use the Laplace transform to solve the following initial value problems.

$$(a) \begin{cases} y' + y = f(t), \\ y(0) = 0, \\ f(t) = \begin{cases} 1, & t \in [0, 1) \\ -1, & t \in [1, \infty) \end{cases} \end{cases} \quad (b) \begin{cases} y'' + 4y = f(t), \\ y(0) = 0, y'(0) = -1, \\ f(t) = \begin{cases} 1, & t \in [0, 1) \\ 0, & t \in [1, \infty) \end{cases} \end{cases} \quad (c) \begin{cases} y'' - 5y' + 6y = H(t - 1), \\ y(0) = 0, y'(0) = 1 \end{cases}$$

3. (Based on 7.4.2, 7.4.6, and 7.4.8.) Use the formula for the derivative of a Laplace transform to evaluate the given Laplace transforms.

$$(a) \mathcal{L}\{t^3 e^t\} \quad (b) \mathcal{L}\{t^2 \cos t\} \quad (c) \mathcal{L}\{te^{-3t} \cos 3t\}$$

4. (Based on 7.4.20, 7.4.22, and 7.4.28.) Use the formula for the Laplace transform of a convolution to evaluate the following Laplace transforms.

$$(a) \mathcal{L}\{t^2 * te^t\} \quad (b) \mathcal{L}\{e^{2t} * \sin t\} \quad (c) \mathcal{L}\left\{\int_0^t \sin \tau \cos(t - \tau) d\tau\right\}$$

5. (Based on 7.4.31, 7.4.32, and 7.4.34.) Use the formula for the Laplace transform of an integral to evaluate the following inverse Laplace transforms.

$$(a) \mathcal{L}^{-1}\left\{\frac{1}{s(s-1)}\right\} \quad (b) \mathcal{L}^{-1}\left\{\frac{1}{s^2(s-1)}\right\} \quad (c) \mathcal{L}^{-1}\left\{\frac{1}{s(s-a)^2}\right\}$$

For additional practice you should try problems 7.3.49–70, 7.4.1–3, 7.4.6–14, and 7.4.19–34. Those looking for an extra challenge should try 7.4.17–18 and 7.4.35–36. We did not cover transforms of hyperbolic sines and cosines, integral equations, applications, or transforms of periodic functions, so the other problems in this chapter are outside the scope of the course.