MATH281 200610 Sample Final 1

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The following final is from 200510, taught by Dr. R. McIntosh.

1. (10 marks) Solve the differential equation $y' = e^{x+y}$.
2. (12 marks) Find the equation of the curve satisfying
$$(4x + y)\,dx + (4y + x)\,dy = 0$$
and passing through the point $(-2, 1)$.
3. (12 marks) Find a second linearly independent solution of
$$xy'' - y' + 4x^3 y = 0$$
given that $y = \cos(x^2)$ is a solution.
4. (12 marks) Solve the differential equation $y'' + y = \sec x$.
5. (12 marks) Use the power series method to solve the initial value problem
$$(x + 2)y' - 2y = 0, \quad y(0) = 4$$
6. (6 marks) Find the Laplace transform of $f(t) = te^{3t} \cos 2t$.
7. (12 marks) Use Laplace transforms to solve the initial value problem
$$y'' + 4y + 3y = \sin 3t, \quad y(0) = 0, \quad y'(0) = 0.$$
8. (12 marks) Find two independent real-valued solutions of the system
$$\begin{aligned} x'(t) &= x(t) + 2y(t) \\ y'(t) &= y(t) - 2x(t). \end{aligned}$$
9. (12 marks) Find the general solution of the system
$$X'(t) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} X(t).$$