MATH122 200610 Sample Final 2

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The following final is from 200010, taught by Dr. S. K. Kaul. Time: 3 hours.

1. Obtain the general solution to the first order differential equation
$$x\frac{dy}{dx} + 3y + 2x^2 = x^3 + 4x.$$
2. (a) Verify that the functions
$$y_1 = x^2$$
$$y_2 = x^{-1}$$
are solutions of the differential equation
$$x^2y'' - 2y = 0$$
on the interval $x > 0$.
   (b) Prove that y_1 and y_2 are linearly independent.
   (c) Find the solution satisfying the initial conditions
$$y(1) = -2$$
$$y'(1) = -7$$
3. Find a second linearly independent solution of
$$x^2y'' - 2xy' - 4y = 0$$
on the interval $x > 0$ given that $y = x^{-1}$ is a solution.
4. Find a general solution to the differential equation
$$3x^2\frac{d^2y}{dx^2} + 11x\frac{dy}{dx} - 3y = 0.$$
5. Solve the following initial value problem without using the Laplace transform.
$$y'' + 2y' + 2y = 0, \quad y(0) = 2, \quad y'(0) = 1.$$
6. Find the general solution to the equation
$$\frac{d^2y}{dx^2} + y = \tan x$$
on the interval $I = (-\pi/2, \pi/2)$ using variation of parameters.
7. Find the Laplace transform of the following functions.

(a) $e^{-t}t\sin 2t$

(b) $\sin 2t \sin 5t$

8. Find the inverse Laplace transform of

$$F(s) = \frac{s^2 + 9s + 2}{(s-1)^2(s+2)}.$$

9. Solve the initial value problem

$$y'' - 3y' + 2y = \cos t, \quad y(0) = 0, \quad y'(0) = 1$$

using the Laplace transform.

10. Find the general solution of the system

$$X'(t) = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 1 \\ 0 & 2 & 4 \end{bmatrix} X(t).$$