 Math 111-001 HW#1

18. $f(x) = x + \cos x \;\Rightarrow\; f'(x) = 1 - \sin x \geq 0$, with equality only if $x = \frac{\pi}{2} + 2n\pi$. So f is increasing on $\mathbb{R}$, and hence, 1-1.

By inspection, $f(0) = 0 + \cos 0 = 1$, so $f^{-1}(1) = 0$.

20. (a) f is 1-1 because it passes the Horizontal Line Test.

(b) Domain of $f = [-3, 3] =$ Range of f^{-1}. Range of $f = [-1, 3] =$ Domain of f^{-1}.

(c) Since $f(0) = 2$, $f^{-1}(2) = 0$.

(d) Since $f(-1.7) \approx 0$, $f^{-1}(0) = -1.7$.

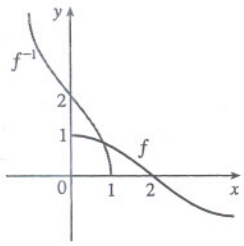
24. $y = f(x) = \dfrac{4x-1}{2x+3} \;\Rightarrow\; y(2x+3) = 4x - 1 \;\Rightarrow\; 2xy + 3y = 4x - 1 \;\Rightarrow\; 3y + 1 = 4x - 2xy \;\Rightarrow$

$3y + 1 = (4 - 2y)x \;\Rightarrow\; x = \dfrac{3y+1}{4-2y}$. Interchange x and y: $y = \dfrac{3x+1}{4-2x}$. So $f^{-1}(x) = \dfrac{3x+1}{4-2x}$.

26. $y = f(x) = 2x^3 + 3 \;\Rightarrow\; y - 3 = 2x^3 \;\Rightarrow\; \dfrac{y-3}{2} = x^3 \;\Rightarrow\; x = \sqrt[3]{\dfrac{y-3}{2}}$. Interchange x and y: $y = \sqrt[3]{\dfrac{x-3}{2}}$.

So $f^{-1}(x) = \sqrt[3]{\dfrac{x-3}{2}}$.

32. Reflect the graph of f about the line $y = x$.



36. (a) $x_1 \neq x_2 \;\Rightarrow\; x_1 - 1 \neq x_2 - 1 \;\Rightarrow\; \dfrac{1}{x_1 - 1} \neq \dfrac{1}{x_2 - 1} \;\Rightarrow\; f(x_1) \neq f(x_2)$, so f is 1-1.

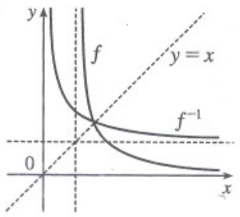
(b) $f^{-1}(2) = \frac{3}{2}$ since $f\left(\frac{3}{2}\right) = 2$. Also $f'(x) = -1/(x-1)^2$, so $(f^{-1})'(2) = 1/f'\left(\frac{3}{2}\right) = \frac{1}{-4} = -\frac{1}{4}$.

(c) $y = 1/(x-1) \;\Rightarrow\; x - 1 = 1/y \;\Rightarrow\; x = 1 + 1/y$. Interchange x and y: $y = 1 + 1/x$. So $f^{-1}(x) = 1 + 1/x$, $x > 0$ (since $y > 1$).

Domain $= (0, \infty)$, range $= (1, \infty)$.

(d) $(f^{-1})'(x) = -1/x^2$, so $(f^{-1})'(2) = -\frac{1}{4}$.

(e)



SECTION 7.2 EXPONENTIAL FUNCTIONS AND THEIR DERIVATIVES

26. Let $t = -x^2$. As $x \to \infty$, $t \to -\infty$. So $\displaystyle\lim_{x\to\infty} e^{-x^2} = \lim_{t\to-\infty} e^t = 0$ by (10).

28. Let $t = 3/(2-x)$. As $x \to 2^-$, $t \to \infty$. So $\displaystyle\lim_{x\to 2^-} e^{3/(2-x)} = \lim_{t\to\infty} e^t = \infty$ by (10).

34. $y = e^u(\cos u + cu) \;\Rightarrow\; y' = e^u(-\sin u + c) + (\cos u + cu)e^u = e^u(\cos u - \sin u + cu + c)$

36. By the Product Rule, $g(x) = \sqrt{x}\, e^x = x^{1/2}e^x \;\Rightarrow\; g'(x) = x^{1/2}(e^x) + e^x\left(\frac{1}{2}x^{-1/2}\right) = \frac{1}{2}x^{-1/2}e^x(2x + 1)$.

38. $f(t) = \sin(e^t) + e^{\sin t} \;\Rightarrow\; f'(t) = \cos(e^t)\cdot e^t + e^{\sin t}\cdot \cos t = e^t\cos(e^t) + e^{\sin t}\cos t$

42. $y = \dfrac{e^u - e^{-u}}{e^u + e^{-u}} \;\Rightarrow$

$$y' = \frac{(e^u + e^{-u})(e^u - (-e^u)) - (e^u - e^{-u})(e^u + (-e^{-u}))}{(e^u + e^{-u})^2} = \frac{e^{2u} + e^0 + e^0 + e^{-2u} - (e^{2u} - e^0 - e^0 + e^{-2u})}{(e^u + e^{-u})^2}$$

$$= \frac{4e^0}{(e^u + e^{-u})^2} = \frac{4}{(e^u + e^{-u})^2}$$