

[was due 22 Jan '10]

SECTION 7.2 EXPONENTIAL FUNCTIONS AND THEIR DERIVATIVES

$$76. \int \frac{(1+e^x)^2}{e^x} dx = \int \frac{1+2e^x+e^{2x}}{e^x} dx = \int (e^{-x} + 2 + e^x) dx = -e^{-x} + 2x + e^x + C$$

$$78. \int e^x (4+e^x)^5 dx \left[\begin{array}{l} u = 4+e^x \\ du = e^x dx \end{array} \right] = \int u^5 du = \frac{1}{6} u^6 + C = \frac{1}{6} (4+e^x)^6 + C$$

$$80. \text{ Let } u = \frac{1}{x}. \text{ Then } du = -\frac{1}{x^2} dx, \text{ so } \int \frac{e^{1/x}}{x^2} dx = -\int e^u du = -e^u + C = -e^{1/x} + C.$$

SECTION 7.3 LOGARITHMIC FUNCTIONS

$$60. y = f(x) = \frac{e^x}{1+2e^x} \Rightarrow y + 2ye^x = e^x \Rightarrow y = e^x - 2ye^x \Rightarrow y = e^x(1-2y) \Rightarrow e^x = \frac{y}{1-2y} \Rightarrow$$

$x = \ln\left(\frac{y}{1-2y}\right)$. Interchange x and y : $y = \ln\left(\frac{x}{1-2x}\right)$. So $f^{-1}(x) = \ln\left(\frac{x}{1-2x}\right)$. Note that the range of f and the domain of f^{-1} is $(0, \frac{1}{2})$.

66. Let (a, e^{-a}) be the point where the tangent meets the curve. The tangent has slope $-e^{-a}$ and is perpendicular to the line $2x - y = 8$, which has slope 2. So $-e^{-a} = -\frac{1}{2} \Rightarrow e^{-a} = \frac{1}{2} \Rightarrow e^a = 2 \Rightarrow a = \ln(e^a) = \ln 2$. Thus, the point on the curve is $(\ln 2, \frac{1}{2})$ and the equation of the tangent is $y - \frac{1}{2} = -\frac{1}{2}(x - \ln 2)$ or $x + 2y = 1 + \ln 2$.

SECTION 7.4 DERIVATIVES OF LOGARITHMIC FUNCTIONS

$$6. f(x) = \log_5(xe^x) \Rightarrow f'(x) = \frac{1}{xe^x \ln 5} \frac{d}{dx}(xe^x) = \frac{1}{xe^x \ln 5} (xe^x + e^x \cdot 1) = \frac{e^x(x+1)}{xe^x \ln 5} = \frac{x+1}{x \ln 5}$$

Another solution: We can change the form of the function by first using logarithm properties.

$$f(x) = \log_5(xe^x) = \log_5 x + \log_5 e^x \Rightarrow f'(x) = \frac{1}{x \ln 5} + \frac{1}{e^x \ln 5} \cdot e^x = \frac{1}{x \ln 5} + \frac{1}{\ln 5} \text{ or } \frac{1+x}{x \ln 5}$$

$$14. F(y) = y \ln(1+e^y) \Rightarrow F'(y) = y \cdot \frac{1}{1+e^y} \cdot e^y + \ln(1+e^y) \cdot 1 = \frac{ye^y}{1+e^y} + \ln(1+e^y)$$

$$18. y = 10^{\tan \theta} \Rightarrow y' = 10^{\tan \theta} (\ln 10) (\sec^2 \theta)$$

$$30. y = \ln(\sec x + \tan x) \Rightarrow y' = \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x} = \sec x \Rightarrow y'' = \sec x \tan x$$

$$70. \int_1^2 \frac{4+u^2}{u^3} du = \int_1^2 (4u^{-3} + u^{-1}) du = \left[\frac{4}{-2} u^{-2} + \ln|u| \right]_1^2 = \left[\frac{-2}{u^2} + \ln u \right]_1^2 = \left(-\frac{1}{2} + \ln 2 \right) - \left(-2 + \ln 1 \right) = \frac{3}{2} + \ln 2$$

$$72. \int_4^9 \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 dx = \int_4^9 \left(x + 2 + \frac{1}{x} \right) dx = \left[\frac{1}{2} x^2 + 2x + \ln x \right]_4^9 = \frac{81}{2} + 18 + \ln 9 - (8 + 8 + \ln 4) = \frac{85}{2} + \ln \frac{9}{4}$$

$$74. \text{ Let } u = \ln x. \text{ Then } du = (1/x) dx, \text{ so } \int \frac{\sin(\ln x)}{x} dx = \int \sin u du = -\cos u + C = -\cos(\ln x) + C.$$

76. Let $u = 2 + \sin x$. Then $du = \cos x dx$, so

$$\int \frac{\cos x}{2 + \sin x} dx = \int \frac{1}{u} du = \ln|u| + C = \ln|2 + \sin x| + C = \ln(2 + \sin x) + C \quad [\text{since } 2 + \sin x > 0].$$

$$78. \text{ Let } u = e^x + 1. \text{ Then } du = e^x dx, \text{ so } \int \frac{e^x}{e^x + 1} dx = \int \frac{du}{u} = \ln|u| + C = \ln(e^x + 1) + C.$$