Math 111-001 HW#3



**22.** $y = \sqrt{\tan^{-1} x} = (\tan^{-1} x)^{1/2} \quad \Rightarrow$

$$y' = \tfrac{1}{2}(\tan^{-1} x)^{-1/2} \cdot \frac{d}{dx}(\tan^{-1} x) = \frac{1}{2\sqrt{\tan^{-1} x}} \cdot \frac{1}{1+x^2} = \frac{1}{2\sqrt{\tan^{-1} x}\,(1+x^2)}$$

**24.** $f(x) = x \ln(\arctan x) \quad \Rightarrow \quad f'(x) = x \cdot \dfrac{1}{\arctan x} \cdot \dfrac{1}{1+x^2} + \ln(\arctan x) \cdot 1 = \dfrac{x}{(1+x^2)\arctan x} + \ln(\arctan x)$

**26.** $g(x) = \sqrt{x^2-1}\,\sec^{-1} x \quad \Rightarrow \quad g'(x) = \sqrt{x^2-1} \cdot \dfrac{1}{x\sqrt{x^2-1}} + \sec^{-1} x \cdot \dfrac{1}{2}(x^2-1)^{-1/2}(2x) = \dfrac{1}{x} + \dfrac{x \sec^{-1} x}{\sqrt{x^2-1}}$

$\left[\text{or } \dfrac{\sqrt{x^2-1} + x^2 \sec^{-1} x}{x\sqrt{x^2-1}}\right]$

**30.** $y = \arctan\sqrt{\dfrac{1-x}{1+x}} = \arctan\left(\dfrac{1-x}{1+x}\right)^{1/2} \quad \Rightarrow$

$$y' = \frac{1}{1+\left(\sqrt{\frac{1-x}{1+x}}\right)^2} \cdot \frac{d}{dx}\left(\frac{1-x}{1+x}\right)^{1/2} = \frac{1}{1+\frac{1-x}{1+x}} \cdot \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-1/2} \cdot \frac{(1+x)(-1)-(1-x)(1)}{(1+x)^2}$$

$$= \frac{1}{\frac{1+x}{1+x}+\frac{1-x}{1+x}} \cdot \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{1/2} \cdot \frac{-2}{(1+x)^2} = \frac{1+x}{2} \cdot \frac{1}{2} \cdot \frac{(1+x)^{1/2}}{(1-x)^{1/2}} \cdot \frac{-2}{(1+x)^2}$$

$$= \frac{-1}{2(1-x)^{1/2}(1+x)^{1/2}} = \frac{-1}{2\sqrt{1-x^2}}$$

**60.** Let $u = \tan^{-1} x$. Then $du = dx/(1+x^2)$, so $\displaystyle\int \frac{\tan^{-1} x}{1+x^2}\,dx = \int u\,du = \tfrac{1}{2}u^2 + C = \tfrac{1}{2}(\tan^{-1} x)^2 + C.$

**62.** Let $u = 2t$. Then $\sqrt{1-4t^2} = \sqrt{1-u^2}$ and $du = 2\,dt$, so

$$\int \frac{dt}{\sqrt{1-4t^2}} = \int \frac{\frac{1}{2}\,du}{\sqrt{1-u^2}} = \frac{1}{2}\sin^{-1} u + C = \frac{1}{2}\sin^{-1}(2t) + C.$$

**64.** Let $u = -\cos x$. Then $du = \sin x\,dx$, so

$$\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}\,dx = \int_{-1}^{0} \frac{1}{1+u^2}\,du = \left[\tan^{-1} u\right]_{-1}^{0} = \tan^{-1} 0 - \tan^{-1}(-1) = 0 - \left(-\tfrac{\pi}{4}\right) = \tfrac{\pi}{4}.$$

**70.** Let $u = x^2$. Then $du = 2x\,dx$, so $\displaystyle\int \frac{x}{1+x^4}\,dx = \int \frac{\frac{1}{2}\,du}{1+u^2} = \tfrac{1}{2}\tan^{-1} u + C = \tfrac{1}{2}\tan^{-1}(x^2) + C.$



**66.** Let $u = e^x$. Then $du = e^x\,dx$, so $\int e^x \sin(e^x)\,dx = \int \sin u\,du = -\cos u + C = -\cos(e^x) + C.$

**68.** Let $u = ax + b$. Then $du = a\,dx$ and $dx = (1/a)\,du$, so

$$\int \frac{dx}{ax+b} = \int \frac{(1/a)\,du}{u} = \frac{1}{a}\int \frac{1}{u}\,du = \frac{1}{a}\ln|u| + C = \frac{1}{a}\ln|ax+b| + C.$$

**70.** Let $u = \cos t$. Then $du = -\sin t\,dt$ and $\sin t\,dt = -du$, so $\int e^{\cos t}\sin t\,dt = \int e^u(-du) = -e^u + C = -e^{\cos t} + C.$

**72.** Let $u = \tan^{-1} x$. Then $du = \dfrac{dx}{1+x^2}$, so $\displaystyle\int \frac{\tan^{-1} x}{1+x^2}\,dx = \int u\,du = \frac{u^2}{2} + C = \frac{(\tan^{-1} x)^2}{2} + C.$

**74.** Let $u = \ln x$. Then $du = (1/x)\,dx$, so $\displaystyle\int \frac{\sin(\ln x)}{x}\,dx = \int \sin u\,du = -\cos u + C = -\cos(\ln x) + C.$

**80.** Let $u = -x^2$, so $du = -2x\,dx$. When $x = 0$, $u = 0$; when $x = 1$, $u = -1$. Thus,

$\int_0^1 xe^{-x^2}\,dx = \int_0^{-1} e^u\left(-\tfrac{1}{2}\,du\right) = -\tfrac{1}{2}\left[e^u\right]_0^{-1} = -\tfrac{1}{2}(e^{-1} - e^0) = \tfrac{1}{2}(1 - 1/e).$