Math 111-001
HW #7

8. $\displaystyle\int_0^\infty \frac{x}{(x^2+2)^2}\,dx = \lim_{t\to\infty}\int_0^t \frac{x}{(x^2+2)^2}\,dx = \lim_{t\to\infty}\frac{1}{2}\left[\frac{-1}{x^2+2}\right]_0^t = \frac{1}{2}\lim_{t\to\infty}\left(\frac{-1}{t^2+2}+\frac{1}{2}\right)$

$= \frac12\left(0+\frac12\right) = \frac14.$ Convergent

14. $\displaystyle\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx = \lim_{t\to\infty}\int_1^t \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx = \lim_{t\to\infty}\int_1^{\sqrt{t}} e^{-u}(2\,du)$ $\quad\left[\begin{array}{l} u=\sqrt{x}, \\ du = dx/(2\sqrt{x}) \end{array}\right]$

$\displaystyle = 2\lim_{t\to\infty}\left[-e^{-u}\right]_1^{\sqrt{t}} = 2\lim_{t\to\infty}\left(-e^{-\sqrt{t}}+e^{-1}\right) = 2(0+e^{-1}) = 2e^{-1}.$ Convergent

18. $\displaystyle\int_0^\infty \frac{dz}{z^2+3z+2} = \lim_{t\to\infty}\int_0^t\left[\frac{1}{z+1}-\frac{1}{z+2}\right]dz = \lim_{t\to\infty}\left[\ln\left(\frac{z+1}{z+2}\right)\right]_0^t$

$\displaystyle = \lim_{t\to\infty}\left[\ln\left(\frac{t+1}{t+2}\right)-\ln\left(\frac12\right)\right] = \ln 1 + \ln 2 = \ln 2.$ Convergent

24. $\displaystyle\int_0^\infty \frac{e^x}{e^{2x}+3}\,dx = \lim_{t\to\infty}\int_0^t \frac{e^x}{(e^x)^2+(\sqrt3)^2}\,dx = \lim_{t\to\infty}\left[\frac{1}{\sqrt3}\arctan\frac{e^x}{\sqrt3}\right]_0^t = \frac{1}{\sqrt3}\lim_{t\to\infty}\left(\arctan\frac{e^t}{\sqrt3}-\arctan\frac{1}{\sqrt3}\right)$

$\displaystyle = \frac{1}{\sqrt3}\left(\frac{\pi}{2}-\frac{\pi}{6}\right) = \frac{1}{\sqrt3}\left(\frac{\pi}{3}\right) = \frac{\pi\sqrt3}{9}.$ Convergent

28. $\displaystyle\int_2^3 \frac{1}{\sqrt{3-x}}\,dx = \lim_{t\to3^-}\int_2^t (3-x)^{-1/2}\,dx = \lim_{t\to3^-}\left[-2(3-x)^{1/2}\right]_2^t = -2\lim_{t\to3^-}\left(\sqrt{3-t}-\sqrt1\right) = -2(0-1) = 2.$
Convergent

30. $\displaystyle\int_6^8 \frac{4}{(x-6)^3}\,dx = \lim_{t\to6^+}\int_t^8 4(x-6)^{-3}\,dx = \lim_{t\to6^+}\left[-2(x-6)^{-2}\right]_t^8 = -2\lim_{t\to6^+}\left[\frac{1}{2^2}-\frac{1}{(t-6)^2}\right] = \infty.$ Divergent

32. $\displaystyle\int_0^1 \frac{dx}{\sqrt{1-x^2}} = \lim_{t\to1^-}\int_0^t \frac{dx}{\sqrt{1-x^2}} = \lim_{t\to1^-}\left[\sin^{-1}x\right]_0^t = \lim_{t\to1^-}\sin^{-1}t = \frac{\pi}{2}.$ Convergent

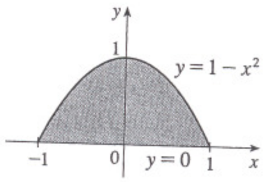
6.2 Volumes

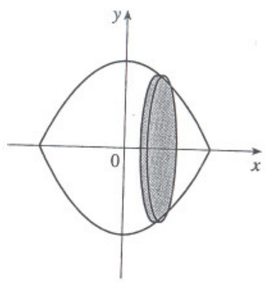
2. A cross-section is a disk with radius $1-x^2$, so its area is $A(x) = \pi(1-x^2)^2$.

$\displaystyle V = \int_{-1}^1 A(x)\,dx = \int_{-1}^1 \pi(1-x^2)^2\,dx$

$\displaystyle = 2\pi\int_0^1 (1-2x^2+x^4)\,dx = 2\pi\left[x-\tfrac23x^3+\tfrac15x^5\right]_0^1$

$= 2\pi\left(1-\frac23+\frac15\right) = 2\pi\left(\frac{8}{15}\right) = \frac{16}{15}\pi$





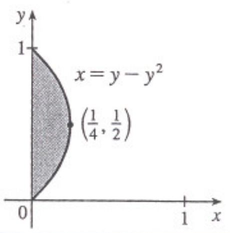
6. A cross-section is a disk with radius $y-y^2$, so its area is $A(y) = \pi(y-y^2)^2$.

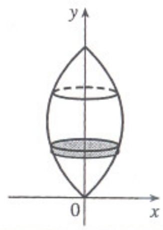
$V = \int_0^1 A(y)\,dy = \int_0^1 \pi(y-y^2)^2dy$

$= \pi(y^4-2y^3+y^2)\,dy = \pi\left[\frac15y^5-\frac12y^4+\frac13y^3\right]_0^1$

$= \pi\left(\frac15-\frac12+\frac13\right) = \frac{\pi}{30}$

10. on side →





18. For $0 \le y < 2$, a cross-section is an annulus with inner radius $2-1$ and outer radius $4-1$, the area of which is $A_1(y) = \pi(4-1)^2-\pi(2-1)^2$. For $2\le y\le 4$, a cross-section is an annulus with inner radius $y-1$ and outer radius $4-1$, the area of which is $A_2(y) = \pi(4-1)^2-\pi(y-1)^2$.

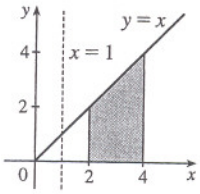
$V = \int_0^4 A(y)\,dy = \pi\int_0^2\left[(4-1)^2-(2-1)^2\right]dy + \pi\int_2^4\left[(4-1)^2-(y-1)^2\right]dy$

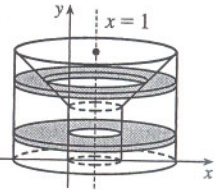
$= \pi\left[8y\right]_0^2 + \pi\int_2^4(8+2y-y^2)\,dy$

$= 16\pi + \pi\left[8y+y^2-\frac13y^3\right]_2^4$

$= 16\pi + \pi\left[\left(32+16-\frac{64}{3}\right)-\left(16+4-\frac83\right)\right]$

$= \frac{76}{3}\pi$





10. A cross-section is a washer with inner radius $x = 2\sqrt{y}$ and outer radius 2, so its area is

$A(y) = \pi\left[(2)^2-\left(2\sqrt{y}\right)^2\right]$

$= \pi(4-4y) = 4\pi(1-y).$

$V = \int_0^1 A(y)\,dy = \int_0^1 4\pi(1-y)\,dy$

$= 4\pi\left[y-\frac12y^2\right]_0^1 = 4\pi\left[\left(1-\frac12\right)-0\right] = 2\pi$